B1/K/TX

Reg. No:

VIT
Vellore Institute of Technology

Final Assessment Test – November 2024

Course: IMAT201L - Complex Variables and Linear Algebra

Slot: C1+TC1+TCC1

Class NBR(s): 2526 / 2527 / 2528 / 2529 / 2530

Max. Marks: 100

Time: Three Hours

- KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- DON'T WRITE ANYTHING ON THE QUESTION PAPER

Answer ALL Questions

(10 X 10 = 100 Marks)

1.(a) Show that the function $u=\frac{1}{2}\log(x^2+y^2)$ is harmonic and find its conjugate and analytic function $f(z)$. [10]

OR

1.(b) In a two dimensional flow of a fluid, the velocity function is $\phi(x,\ y)=x^2-y^2$, find the stream function $\psi(x,\ y)$. [10]

2. Find the image of the infinite strip $\frac{1}{4}\le y\le\frac{1}{2}$ in the z-plane under the transformation $w=\frac{1}{z}$. [10]

3. Find the bilinear transformation which maps the points $1,\ i,\ -1$ in the z-plane into the points $2,\ i,\ -2$ into the w-plane. Also find the invariant points of it. [10]

4. Obtain the Laurent series expansion of the function $f(z)=\frac{7z-2}{z(z+1)(z-2)}$ in the region $1<|z+1|<3$. [10]

5. Evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)}dz$ where C is the $|z|=\frac{3}{2}$ using residue theorem. [10]

6. Find the basis for row space, column space and null space for [10]

$$A=\begin{pmatrix}1 & -2 & 0 & 0 & 3\\ 2 & -5 & -3 & -2 & 6\\ 0 & 5 & 15 & 10 & 0\\ 2 & 6 & 18 & 8 & 6\end{pmatrix}.$$

7.(a) Let $\beta=\{v_1,v_2,v_3\}$ be the basis for R^3, where $v_1=(1,1,1)$, $v_2=(1,1,0)$, $v_3=(1,0,0)$ and $T:R^3(R)\to R^2(R)$ be the linear transformation defined by $T(1,1,1)=(1,0)$, $T(1,1,0)=(2,-1)$, $T(1,0,0)=(4,3)$ find a formula for $T(x,y,z)$ and then compute $T(2,-3,5)$. [10]

OR

7.(b) Let α be the standard basis of R^3 and let $S,T: R^3 \to R^3$ be two linear transformation given by [10]

$S(e_1)=(2,2,1),\ S(e_2)=(0,1,2),\ S(e_3)=(-1,2,1)$

$T(e_1)=(1,0,1),\ T(e_2)=(0,1,1),\ T(e_3)=(1,1,2)$

Compute $[S+T]_\alpha$ and $[2T-S]_\alpha$.

8. Obtain an orthogonal basis for the subspace of $R^3(R)$ spanned by $\alpha_1=(2,\ 1,\ 3),\ \alpha_2=(1,\ 2,\ 3),\ \alpha_3=(1,\ 1,\ 1)$ using Gram Schmidt orthogonal process. [10]

9. Obtain the eigenvalues and the corresponding eigenvectors of $A=\begin{pmatrix}3 & -1 & 3\\ 9 & -1 & 9\\ 7 & -1 & 7\end{pmatrix}$. [10]

10. Solve the system of linear equations [10]

$$\begin{aligned} x_1+3x_2-2x_3 &= 3\\ 2x_1+6x_2-2x_3+4x_4 &= 18\\ x_2+x_3+3x_4 &= 10 \end{aligned}$$

by Gauss Jordan method.

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