FAT Model Question Paper

Fall Sem. 2024-25

Programme Name & Branch: B. Tech.

Course Name & Code: Complex Variables and Linear Algebra (BMAT201L)

Maximum Time: 180 minutes **Maximum Marks: 100**

Answer all the questions

(Each question carries 10 marks)

1. If $f(z) = u + iv$ is an analytic function of z and $u - v = e^x(\cos y - \sin y)$, find $f(z)$ in terms of z.

OR

If $w = f(z) = \phi + i\psi$ represents the complex potential function for an electric field and $\psi(x,\ y) = x^2 - y^2 + \dfrac{x}{x^2 + y^2}$, then find $\phi(x,\ y)$.

2. Find the image of the region bounded by the lines $x = 1,\ y = 1$ and $x + y = 1$ in the z-plane under the transformation $w = z^2$.

3. Obtain the bilinear transformation which maps the points $0,\ i,\ 1$ in the z-plane into the points $-1,\ 0,\ 1$ into the w-plane.

4. Obtain the Laurent series expansion of the function $f(z) = \dfrac{z^2 - 6z - 1}{(z-1)(z-3)(z+2)}$ in the region $3 < |z + 2| < 5$.

5. Evaluate $\displaystyle\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta}\, d\theta$ using Residue theorem.

6. a) Find the basis and dimension of the subspace $\text{W}=\{(\text{x, y, z, w}) \mid \text{x+y=0, z=2w}\}$ of a vector space $\text{R}^4(\text{R})$. [5 M]

b) Find the nullity of the matrix $A = \begin{pmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 1 \end{pmatrix}$. [5 M]

7. Let $T: R^2(R) \to R^3(R)$ be a linear transformation defined by $T(x, y) = (x+y, x-y, y)$. Find the basis and dimension of the range space and the null space of T.

OR

Let $T: R^3(R) \to R^2(R)$ be a linear transformation defined by $T(x, y, z) = (2x+y-z, 3x-2y+4z)$. Find the matrix of the linear transformation T with respect to the bases $B_1 = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ and $B_2 = \{(1, 3), (1, 4)\}$.

8. Using Gram Schmidt process, find an orthogonal basis for $R^3(R)$, given that $\{(1, 2, 2), (-1, 0, 2), (0, 0, 1)\}$ is a basis for R^3 and hence find an orthonormal basis for $R^3(R)$.

9. Let $A = \begin{pmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ -3 & 4 & 6 \end{pmatrix}$. Using Cayley-Hamilton theorem, find A^{-1} and A^4.

10. Solve the following system of equations using Gauss Jordan method

$$x - y + 2z - 3w = -1;\ 2x + y - 3z + w = 1;\ 3x - 2y + z + 2w = 4;\ x + 3y - z + 4w = 7.$$