VIT®

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

School of Advanced Sciences

Fall Semester 2024-2025

Continuous Assessment Test – I

Programme Name & Branch: B.Tech. & ALL
Slot : A1+TA1+TAA1
Course Name & code : Complex Variables and Linear Algebra & BMAT201L
Class Number (s):VL2024250102491, 2492, 2500, 2501, 2503, 2505, 2493, 2506, 2495, 2494, 2497, 2498, 2499, 2489, 2496

Exam Duration: 90 Min. **Maximum Marks: 50**

General instruction(s) **Answer ALL questions (5x10=50 Marks)**

Q.No.	Question	Max Marks
1.	Determine the analytic function $f(z) = u + iv,$ given that $3u + 2v = y^2 - x^2 + 16xy$.	10
2.	Show that $\psi(x, y) = x^2 - y^2 - 3x - 2y + 2xy$ can represent the stream function of an incompressible fluid flow. Also find the corresponding velocity potential and complex potential.	10
3.	If the points $1, i, -1$ in the z-plane are mapped onto the points $i, 0, -i$ in the *w*-plane respectively, then (i) Find the corresponding bilinear transformation $w = f(z)$. (ii) Find the invariant points of this transformation.	10
4.	Show that the transformation $w = \frac{5-4z}{4z-2}$ maps the circle |z|=1 into a circle of radius unity in *w*-plane and find the centre of the circle.	10
5.	Express the function $f(z) = \frac{(z-2)\,(z+2)}{(z+1)\,(z+4)}$ in Laurent series about (i) $1 < \|z\| < 4$ (ii) $\|z\| > 4$	10

Fall Semester 2024-25

CAT-I

BMAT201L - Complex Variables and Linear Algebra

Slot : A1+TA1+TAA1

(1) $3u + 2v = y^2 - x^2 + 16xy$

$3u_x + 2v_x = -2x + 16y$ --- (1)

$3u_y + 2v_y = 2y + 16x$ --- (2)

by C-R equ.

$u_x = 2x + 4y$; $v_x = -4x + 2y$

$f'(z) = u_x + i v_x$

$= 2z - i4z$. by (M-T)

$f(z) = (1 - i2) z^2 + c$.

(2) $\phi(x, y) = x^2 - y^2 - 3x - 2y + 2xy$

$\phi_{xx} = 2$; $\phi_{yy} = -2$

$\therefore$ ϕ is a harmonic function.

Let $F(z) = \phi + i\psi$ is analytic.

By C-R $\phi_x = \psi_y$ and $\phi_y = -\psi_x$

$$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy = \frac{\partial \psi}{\partial y} dx - \frac{\partial \psi}{\partial x} dy$$

$$d\phi = (-2y - 2 + 2x)\,dx + (2x - 3 + 2y)\,dy$$

$$\therefore \phi = x^2 - y^2 - 2x + 3y - 2xy + c$$

$$F(z) = [x^2 - y^2 - 2x + 3y - 2xy + c] + i[x^2 - y^2 - 3x - 2y + 2xy]$$

by M–T

The complex potential

$$F(z) = z^2 - 2z + c + i[z^2 - 3z]$$

(3) Let $z_1 = \cancel{\#}1$, $z_2 = i$, $z_3 = -1$, $z_4 = z$

$w_1 = i$, $w_2 = 0$, $w_3 = -i$, $w_4 = w$

the bilinear transformation

$$w = \frac{1 + iz}{1 - iz}.$$

The invariant points of the transformation

$$w = \frac{1+iz}{1-iz} \quad ; \; w = z$$

$$iz^2 + z(i-1) + 1 = 0.$$

$$z = \frac{-(i-1) \pm \sqrt{-6i}}{2i}$$

(4) $$w = \frac{5-4z}{4z-2} \quad : \quad C : |z| = 1.$$

the inverse of the transformation is

$$z = \frac{2w+5}{4w+4}$$

$|z| = z\bar{z} = 1$ corresponds to

$$\left[\frac{2w+5}{4(w+1)}\right]\left[\frac{2\bar{w}+5}{4(\bar{w}+1)}\right] = 1.$$

(or) $$4w\bar{w} + 25 + 10(w+\bar{w}) = 16(w\bar{w} + w + \bar{w} + 1) \rightarrow (1)$$

$$w\bar{w} = u^2 + v^2 \; ; \; w + \bar{w} = 2u$$

from (1)

$$4(u^2+v^2) + 25 + 10 \cdot 2u.$$

$$= 16\left[u^2+v^2+2u+1\right]$$

(or) $u^2+v^2+u-3/4=0$ --- (2)

$(u+1/2)^2+v^2=1$

centre : $(-1/2, 0)$; radius : 1

(5) $f(z)=\dfrac{(z-2)(z+2)}{(z+1)(z+4)}$

Let $f(z)=1+\dfrac{(-5z-8)}{(z+1)(z+4)}$

$\boxed{\begin{matrix}A=-1\\B=-4\end{matrix}}$

by P.F $f(z)=1-\dfrac{1}{1+z}-\dfrac{4}{z+4}$.

(i) Given $1<|z|<4$

$$f(z)=\sum_{n=1}^{\infty}(-1)^n\left[\frac{1}{z^n}-\left(\frac{z}{4}\right)^n\right]$$

(ii) Given $|z|>4$

$$f(z)=1+\sum_{n=0}^{\infty}(-1)^n\left[\frac{1}{z^{n+1}}+\left(\frac{4}{z}\right)^{n+1}\right]$$

$$=1+\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{z^{n+1}}\left(1+4^{n+1}\right)$$

—— x ——