Reg. No: 23BCA0041

BI/K/TX

Final Assessment Test – November 2024

VIT
Vellore Institute of Technology

Course: BMAT201L - Complex Variables and Linear Algebra

Class NBR(s): 2458 / 2459 / 2460 / 2461 / 2462 / 2463 / 2464 / 2465 / 2466 / 2467 / 2468 / 2470 / 2471 / 8120

Slot: C1+TC1+TCC1

Time: Three Hours

Max. Marks: 100

- KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- DON'T WRITE ANYTHING ON THE QUESTION PAPER

Answer ALL Questions

(10 X 10 = 100 Marks)

1.a) Can $\varphi = (x-y)(x^2+4xy+y^2)$ represent the equipotential for an electric field. If so find the corresponding complex potential $w = \varphi + i\psi$ and also ψ, if possible. **[10]**

OR

1.b) Find the analytic function $w = u + iv$, where $u = e^{-2xy}\sin(x^2-y^2)$. Hence find v. **[10]**

2. Using the transformation $w = \frac{1}{z}$, find the image of the circle $|z-3| = 5$. **[10]**

3. Find the bilinear transformation that maps the points $z_1 = -1,\ z_2 = 0, z_3 = 1$ into the points $\omega_1 = 0, \omega_2 = i, \omega_3 = 3i$ respectively. Also find the invariant points of the transformation. **[10]**

4. Find the Laurent's series expansion of the function $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ which is valid in the range (i) $2 < |z| < 3$, and (ii) $|z| > 3$. **[10]**

5. Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta}\, d\theta$ by contour integration. **[10]**

6. Find the basis and dimension of row space and null space of **[10]**

$$A = \begin{bmatrix} 1 & -1 & 2 & 1 & 0 \\ 2 & 0 & 1 & -1 & 3 \\ 5 & -1 & 3 & 0 & 3 \\ 4 & -2 & 5 & 1 & 3 \\ 1 & 3 & -4 & -5 & 6 \end{bmatrix}.$$

7.a) Let $G : R^3 \to R^3$ be the linear mapping defined by **[10]**

$$G(x,y,z) = (x+2y-z,\ y+z,\ x+y-2z).$$

Find a basis and the dimension of (i) The image of G, (ii) The kernel of G.

OR

7.b) Let $T : R^3 \to R^2$ be the linear transformation defined by **[10]**

$$T(x,y,z) = (2x+y-z;\ 3x-2y+4z)$$

Find $[T]_\alpha^\beta$, for $\alpha = \{(1,1,1),(1,1,0),(1,0,0)\}$ and $\beta = \{(1,3),(1,4)\}$.

8. Let V be the vector space of polynomials $f(t)$ with the inner product $< f.g >= \int_{-1}^{1} f(t)g(t)dt$. Apply the Gram–Schmidt orthogonalization process to $\{1, t, t^2, t^3\}$ to find an orthogonal basis $\{u_1, u_2, u_3, u_4\}$. **[10]**

9. Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$, **[10]**

hence find the eigen values of A^{-1}, $A^T and\ A^4$.

10. Using Gauss-elimination method, solve the system of equations **[10]**

$x + y + z + w = 2$, $x + y + 3z - 2w = -6$, $2x + 3y - z + 2w = 7$,

$x + 2y + z - w = -2$.

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