REG.NO.:

21

# VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

NAME OF THE SCHOOL

CONTINUOUS ASSESSMENT TEST - II

WINTER SEMESTER 2024-2025

SLOT: G2+TG2

| | |
|---|---|
| **Programme Name & Branch** | **: B.Tech, Mechanical Engineering** |
| **Course Code and Course Name** | **: BMEE306L, Computer Aided Design and Finite Element Analysis** |
| **Faculty Name(s)** | **: Dr. Sharan Chandran, Dr Renold Elsen, Dr. Mallikarjuna Reddy D, Dr. Niranjana Behera, Dr. Oyyaravelu R** |
| **Class Number(s)** | **: VL2024250103577** |
| **Date of Examination** | **: 02-02-2024** |
| **Exam Duration** | **: 90 minutes** |

**General instruction(s):** **Maximum Marks: 50**

- Answer All Questions
- M - Max mark; CO – Course Outcome; BL – Blooms Taxonomy Level (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create)
- Course Outcomes
  1. Develop concept model into CAD model using geometric modelling techniques.
  2. Apply suitable product data exchange techniques to convert geometric model into numerical model.
  3. Generate mathematical representation of curves, surfaces and solids using interpolation and approximation concepts.

| Q.No | Question | Marks | CO | BL |
|---|---|---|---|---|
| 1. | A triangle ABC with vertices A(4,7), B(10,15) and C(32,25) is rotated by 45° CW about origin followed by a 5 units and 10 units translation in x and y directions respectively . Determine the concatenated transformation matrix and the new coordinates of the triangle. | 10 | 1 | 3 |
| 2. | A rectangular object has an edge length of 10 units along x-axis and 12 units along y-axis with its centre located at the origin. If the object is scaled about its centre by factor of 3 and 2 along the axes x and y, respectively and subsequently translated by 5 units along x axis, determine the concatenated transformation matrix and calculate the new position of the vertices of the object. | 10 | 1 | 4 |
| 3. | Find the parametric equation of Hermite cubic spline with the end point P0 (3,1) and P1(12,7) whose tangent vector for end are given as P0'(5,6) and P1'(11,8). Evaluate the position and tangent vectors at parameter values t=0.3 and t=0.6 of the curve. | 10 | 3 | 4 |
| 4. | Derive cubic Bezier curve. Find the parametric equation of the Bezier curve whose end control points are $P_0$ (0,0) and $P_3$ (7,4). The other control points are $P_1$ (3,5) $P_2$ (4,6). Evaluate the position vector at a point t=0.6. | 10 | 3 | 4 |

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| | | | | |
|---|---|---|---|---|
| 5. | 1. Explain various standards of graphics exchange in modelling.<br>2. Explain Bicubic Bezier surface with parametric equation | 5<br>5 | 2 | 2 |

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# Winter Semester 2024-25

## CAT I Key

(1)

$x_A = 4 \quad x_B = 10 \quad x_C = 32$

$y_A = 7 \quad y_B = 15 \quad y_C = 25$

$\theta = -45$ (Clockwise)

Transformation matrix for rotation about origin

$$[T_1] = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(-45) & -\sin(-45) & 0 \\ \sin(-45) & \cos(-45) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.707 & +0.707 & 0 \\ -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2

Transformation matrix for translation

$$[T_2] = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix}$$

2

Concatenated matrix

$$[T] = [T_2][T_1]$$

3

$$= \begin{bmatrix} 0.707 & 0.707 & 5 \\ -0.707 & 0.707 & 10 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_A' & x_B' & x_C' \\ y_A' & y_B' & y_C' \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.707 & 0.707 & 5 \\ -0.707 & 0.707 & 10 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_A & x_B & x_C \\ y_A & y_B & y_C \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.707 & 0.707 & 5 \\ -0.707 & 0.707 & 10 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 10 & 32 \\ 7 & 15 & 25 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 12.78 & 22.68 & 45.3 \\ 12.12 & 13.53 & 5.05 \\ 1 & 1 & 1 \end{bmatrix}$$

3 ✓

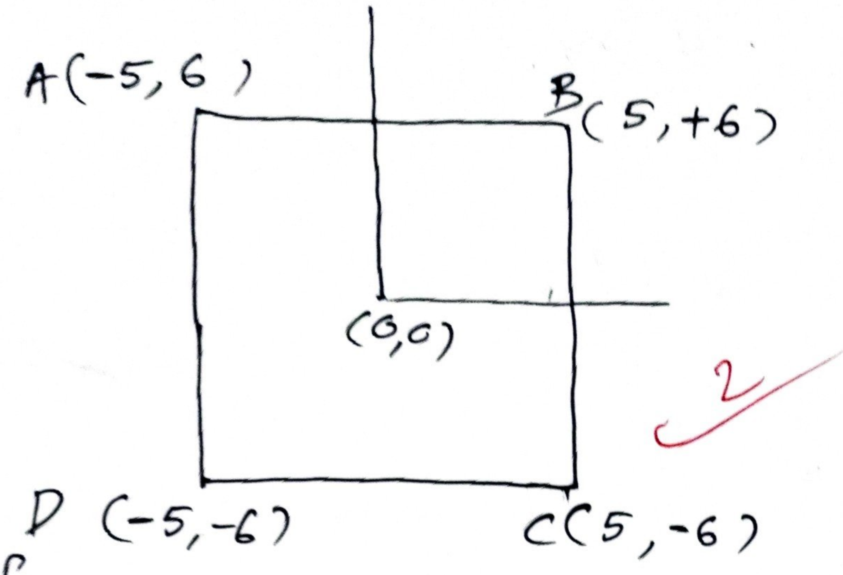
New coordinates

$A'$ $(12.78, 12.12)$

$B'$ $(22.68, 13.53)$

$C'$ $(45.3, 5.05$

(2)




Matrix for
Scaling about its center

$$[T_1] = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix for translation

$$[T_2] = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Concatenated matrix $[T]$ $= [T_2][T_1]$

$$= \begin{bmatrix} 3 & 0 & 5 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_A' & x_B' & x_C' & x_D' \\ y_A' & y_B' & y_C' & y_D' \\ 1 & 1 & 1 & 1 \end{bmatrix} = [T] \begin{bmatrix} x_A & x_B & x_C & x_D \\ y_A & y_B & y_C & y_D \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -10 & 20 & 20 & -10 \\ 12 & 12 & -12 & -12 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

3

$$P_0 = [3 \quad 1]$$

$$P_1 = [12 \quad 7]$$

$$P_0' = [5 \quad 6]$$

$$P_1' = [11 \quad 8]$$

$$P_x(u) = \begin{bmatrix} P_{0x} & P_{1x} & P_{0x}' & P_{1x}' \end{bmatrix} \times \begin{bmatrix} 2 & -3 & 0 & 1 \\ -2 & 3 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u^3 \\ u^2 \\ u \\ 1 \end{Bmatrix}$$

3

$$= \begin{bmatrix} 3 & 12 & 5 & 11 \end{bmatrix} \begin{bmatrix} 2 & -3 & 0 & 1 \\ -2 & 3 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u^3 \\ u^2 \\ u \\ 1 \end{Bmatrix}$$

$$= \begin{bmatrix} -2 & 6 & 5 & 3 \end{bmatrix} \begin{Bmatrix} u^3 \\ u^2 \\ u \\ 1 \end{Bmatrix}$$

$$= -2u^3 + 6u^2 + 5u + 3$$

$$P_y(u) = \begin{bmatrix} P_{0y} & P_{1y} & P'_{0y} & P'_{1y} \end{bmatrix} \begin{bmatrix} 2 & -3 & 0 & 1 \\ -2 & 3 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u^3 \\ u^2 \\ u \\ 1 \end{Bmatrix}$$

$$= \begin{bmatrix} 1 & 7 & 6 & 8 \end{bmatrix} \begin{bmatrix} 2 & -3 & 0 & 1 \\ -2 & 3 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u^3 \\ u^2 \\ u \\ 1 \end{Bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & 6 & 1 \end{bmatrix} \begin{Bmatrix} u^3 \\ u^2 \\ u \\ 1 \end{Bmatrix}$$ 3

$$= 2u^3 - 2u^2 + 6u + 1$$

$$P'_x(u) = -6u^2 + 12u + 5$$ 1

$$P'_y(u) = 6u^2 - 4u + 6$$

4

| | $u = 0.3$ | $u = 0.6$ |
|---|---|---|
| $P_x(u)$ | 4.986 | ~~8.16~~ 7.728 |
| $P_y(u)$ | 2.674 ~~0.036~~ | ~~-2.88~~ 4.312 |
| $P'_x(u)$ | 8.06 | 10.04 |
| $P'_y(u)$ | ~~6.6~~ 5.34 | ~~-14.4~~ 5.76 |

1 1

(4) for a cubic Bezier curve

$n = 3$

$$P(u) = \sum_{i=0}^{n} P_i \, B_{i,n}(u)$$

$$= \sum_{i=0}^{3} P_i \, B_{i,n}(u)$$

$$= P_0 B_{0,3} + P_1 B_{1,3} + P_2 B_{2,3} + P_3 B_{3,3}$$

$$B_{i,n} = C(n,i)\, u^i (1-u)^{n-i}$$

$$B_{0,3} = C(3,0)\, u^0 (1-u)^{3-0}$$

$$= \frac{3!}{0!\,(3-0)!} \, 1 \times (1-u)^3$$ 4

$$= (1-u)^3$$

$$B_{1,3} = C(3,1)\, u^1 (1-u)^{3-1}$$

$$= \frac{3!}{1!\,(3-1)!} \, u^1 (1-u)^2$$

$$= 3u(1-u)^2$$

$$B_{2,3} = C(3,2)\, u^2 (1-u)^{3-2}$$

$$= \frac{3!}{2!\,(3-2)!} \, u^2 (1-u)$$

$$= 3u^2(1-u)$$

$$B_{3,3} = C(3,3)\, u^3 (1-u)^{3-3}$$

$$= \frac{3!}{3!\,(3-3)!} \, u^3$$

$$= u^3$$

$$P(u) = P_0(1-u)^3 + P_1 3u(1-u)^2 + P_2 3u^2(1-u) + P_3 u^3$$

At $u = 0.6$,

$$P_x = P_{0x}(1-u)^3 + P_{1x} 3u(1-u)^2 + P_{2x} 3u^2(1-u) + P_{3x} u^3$$

2

$$= 0 \times (1-0.6)^3 + 3 \times 3 \times 0.6(1-0.6)^2 + 4 \times 3 \times 0.6^2(1-0.6) + 7 \times 0.6^3$$

1

$$= \cancel{5.112} \; \cancel{5.184} \; 4.104$$

8

$$P_y = P_{0y}(1-u)^3 + P_{1y} 3u(1-u)^2 + P_{2y} 3u^2(1-u) + P_{3y} u^3$$

2

$$= 0(1-u)^3 + 5 \times 3 \times 0.6(1-0.6)^2 + 6 \times 3 \times 0.6^2(1-0.6) + 4 \times 0.6^3$$

1

$$= \cancel{5.472} \; \cancel{4.896} \; 4.896$$