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SCHOOL OF MECHANICAL ENGINEERING
CONTINUOUS ASSESSMENT TEST – II
WINTER SEMESTER 2024-2025

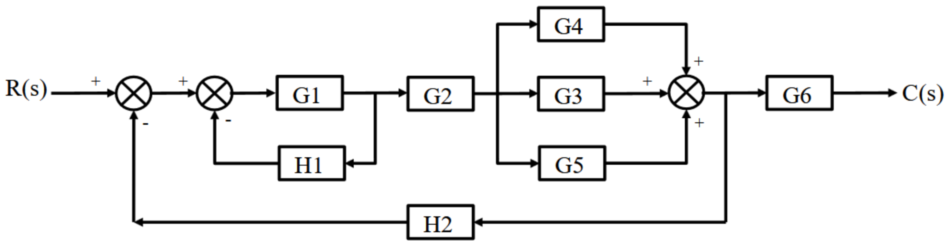
Program Name & Branch : **B. Tech – BMA, BME, BMM**
Course Code : **BMEE330L**
Course Name : **Control Systems**
Faculty Name(s) : **Prof. Vasudevan R, Prof. Giriraj M, Prof. Sathish G.P., Prof. Sovan Sundar D**
Class Number(s) : **VL2024250503927, XXXX, XXXX, XXXX**
Date of Examination :
Maximum Marks: 50 **Exam Duration: 90 mins**

General instruction(s): OPEN BOOK Examination

- Answer All Questions
- M - Max mark; CO – Course Outcome; BL – Blooms Taxonomy Level (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create)
- Course Outcomes
  CO. 1: Apply the concepts of control systems and modelling techniques.
  CO. 3: Infer the domain specifications from the time and frequency response.
  CO. 4: Analyse the stability of closed-loop systems using different techniques.

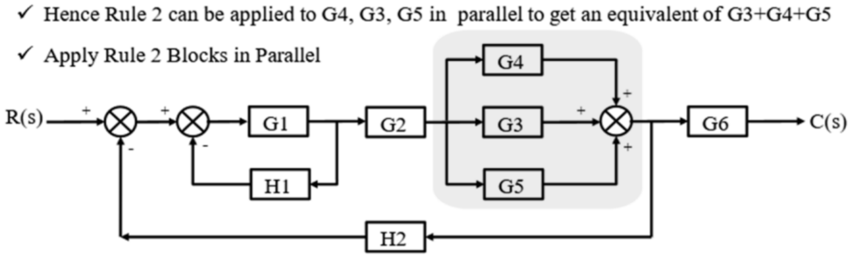
KEY

Q. No	Question
1.	Using the block reduction technique find the transfer function $\frac{C(S)}{R(S)}$ for the system shown in the Figure below.



Solution:

✓ Rule 1 cannot be used as there are no immediate series blocks.
✓ Hence Rule 2 can be applied to G4, G3, G5 in parallel to get an equivalent of G3+G4+G5
✓ Apply Rule 2 Blocks in Parallel



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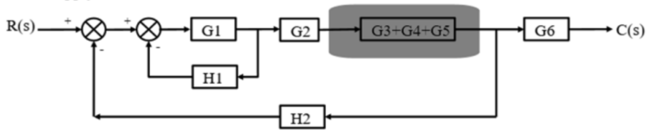
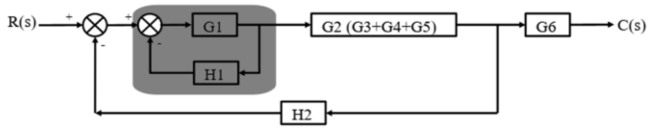
<table>
<tr><td></td><td>

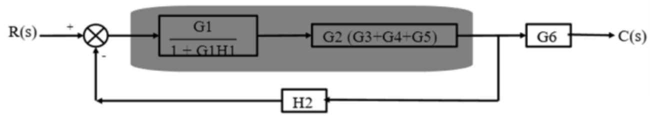
✓ Apply Rule 1 Block in Series



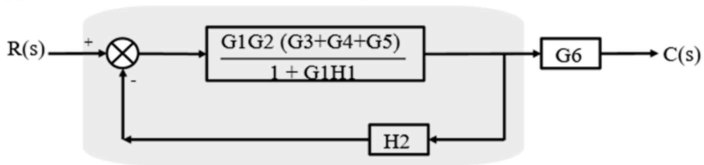
✓ Apply Rule 3 Elimination of feedback



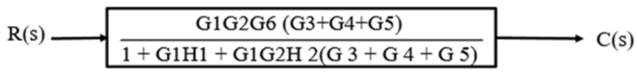
✓ Apply Rule 1 Block in Series



✓ Apply Rule 3 Elimination of feedback loop



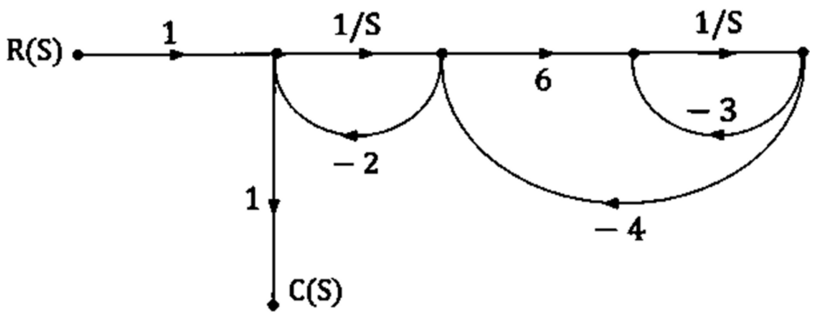
✓ Apply Rule 1 Block in Series



$$\frac{C(s)}{R(s)} = \frac{G1G2G6\ (G3+G4+G5)}{1 + G1H1 + G1G2H\ 2(G\ 3 + G\ 4 + G\ 5)}$$

</td></tr>
<tr><td>2.</td><td>

For the Signal Flow Graph shown in the Figure below determine $\frac{C(S)}{R(S)}$ using Mason's gain formula.



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Solution: Number of forward paths = 1

Forward path is the path from input node to output node without repitition of any node.

Forward path gain = 1

Individual loops: $L_1 = -\frac{2}{S}$ $\quad L_2 = 6 \times (-4) \times \frac{1}{S} = \frac{-24}{S}$ $\quad L_3 = (-3) \times \frac{1}{S}$

Calculation of Δ :

Δ = 1 − [Sum of gains of all the Individual loops]
+ [Sum of Products of gains of all possible combinations of two Non-touching Loops]
− [Sum of products of gains of all possible combinations of three Non-touching Loops] + ...

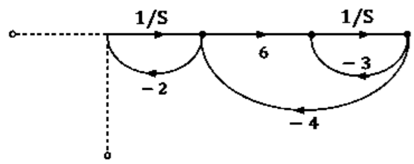
$$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1 . L_3) - (0)$$

$$= 1 - \left(\frac{-2}{S} + \frac{-24}{S} + \frac{-3}{S}\right) + \left(\frac{6}{S^2}\right)$$

$$= 1 + \frac{29}{S} + \frac{6}{S^2}$$

$$\boxed{\Delta = \frac{S^2 + 29.S + 6}{S^2}}$$

Associated path factor:



$$\Delta_1 = 1 - (L_2^* + L_3^*) + (0) = 1 - \left(-\frac{24}{S} - \frac{3}{S}\right) = 1 + \frac{27}{S}$$

$$\boxed{\Delta_1 = \frac{S + 27}{S}}$$

Applying Mason's gain formula:

$$P_1 = 1 \qquad \Delta_1 = \frac{S + 27}{S} \qquad \Delta = \frac{S^2 + 29.S + 6}{S^2}$$

$$\frac{C(S)}{R(S)} = \sum_{n=1}^{1} \frac{P_k . \Delta_k}{\Delta} = \frac{P_1 . \Delta_1}{\Delta}$$

$$= \frac{1 \times \frac{S + 27}{S}}{\frac{S^2 + 29.S + 6}{S^2}}$$

$$\boxed{\frac{C(S)}{R(S)} = \frac{S.(S + 27)}{S^2 + 29.S + 6}}$$

3. Explain the different time domain parameters of 1st order and 2nd order closed loop systems. Explain each parameter with the plots.

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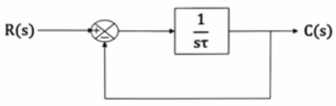
Time Domain Parameters

- These are the parameters that are used to define the performance of control system in the time domain.
- We use step signal as the input to define various time domain parameters both in case of 1st order and 2nd order systems.

Time Domain Parameters of 1st Order Systems

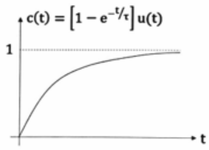
1. Delay time
2. Rise time
3. Settling time

Let us consider the standard 1st order system with step signal as the input:



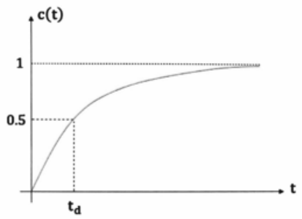
$$\frac{C(s)}{R(s)} = \frac{1}{1+\tau . s}$$

$$c(t) = \left[1 - e^{-t/\tau}\right] u(t)$$



1. Delay time (t_d):

It is defined as the time taken by the response of the system to reach from 0 to 50% of it's steady state value.



$$c(t) = \left[1 - e^{-t/\tau}\right] u(t)$$

$$0.5 = 1 - e^{-t_d/\tau}$$

$$e^{-t_d/\tau} = 0.5$$

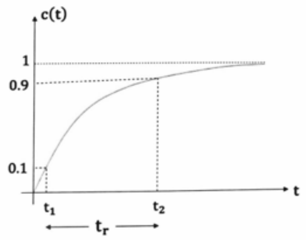
$$-\frac{t_d}{\tau} = \ln(0.5) = -0.693$$

$$\boxed{t_d = 0.693 \times \tau}$$

So, the delay time of 1st order system is $t_d = 0.693\,\tau$

2. Rise time (t_r):

Rise time of a 1st order system is defined as the time taken by response to reach from 10% to 90% of it's steady state value.



t_1: $c(t) = 1 - e^{-t/\tau}$

$$0.1 = 1 - e^{-t_1/\tau}$$

$$\Rightarrow \quad e^{-t_1/\tau} = 0.9$$

$$\Rightarrow \quad -\frac{t_1}{\tau} = \ln(0.9)$$

$$\Rightarrow \quad -\frac{t_1}{\tau} = -0.105$$

$$\Rightarrow \quad t_1 = 0.105\tau$$

t_2: $c(t) = 1 - e^{-t/\tau}$

$$0.9 = 1 - e^{-t_2/\tau}$$

$$\Rightarrow \quad e^{-t_2/\tau} = 0.1$$

$$\Rightarrow \quad -\frac{t_2}{\tau} = \ln(0.1)$$

$$\Rightarrow \quad -\frac{t_2}{\tau} = -2.305$$

$$\Rightarrow \quad t_2 = 2.305\tau$$

Rise time: $t_r = t_2 - t_1 \;\Rightarrow\; t_r = 2.305\tau - 0.105\tau \;\Rightarrow\; \boxed{t_r = 2.2 \times \tau}$

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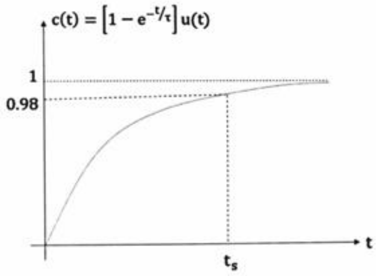
2. Settling time (t_s):

Settling time of a 1st order system is defined as the time taken by response of the system to reach to the steady state value and stay within 2% error band.



$$c(t) = 1 - e^{-t/\tau}$$

$$0.98 = 1 - e^{-t_s/\tau}$$

$$\Rightarrow \quad e^{-t_s/\tau} = 0.02$$

$$\Rightarrow \quad -\frac{t_s}{\tau} = \ln(0.02)$$

$$\Rightarrow \quad -\frac{t_s}{\tau} = -3.912$$

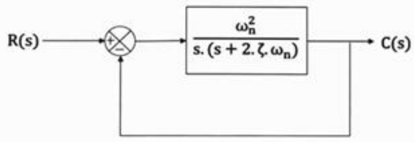
$$\Rightarrow \quad t_s = 3.912\tau$$

Settling time for a 1st order system is: $\boxed{t_s = 4 \times \tau}$

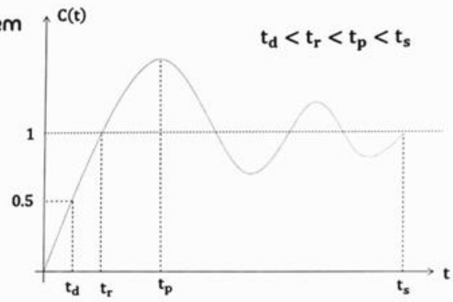
Conclusions:

$$\boxed{\begin{aligned} t_d &= 0.693 \times \tau \\ t_r &= 2.2 \times \tau \\ t_s &= 4 \times \tau \end{aligned}}$$

Time Domain Parameters of 2nd Order System



$$\boxed{c(t) = \left[1 - \frac{e^{-\alpha t}}{\sqrt{1-\zeta^2}}\{\sin(\omega_d t + \theta)\}\right].u(t)}$$



$$t_d < t_r < t_p < t_s$$

- **The Time Domain Parameters are defined for a 2nd order system when the system is underdamped and the input is step signal.**
- **Various Time Domain Parameters of 2nd order system are:**

1. Peak time (t_p)
2. Maximum Peak Overshoot (MPO)
3. Maximum Peak undershoot (MPU)
4. Rise time (t_r)
5. Settling time (t_s)
6. Delay time (t_d)

1. Peak Time (t_p):

- It is the time required for the response to reach it's peak value.
- It is also defined as the time at which the response undergoes it's first overshoot (Peak Overshoot).

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\{\sin(\omega_d t + \theta)\} \;;t \geq 0$$

Applying the concept of maxima & minima to maximize c(t):

$$\frac{d.c(t)}{dt} = 0$$

$$\Rightarrow 0 - \left[\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}.\omega_d \cos(\omega_d t + \theta) + \sin(\omega_d t + \theta).(-\zeta\omega_n).\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\right] = 0$$

$$\Rightarrow \quad \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}.\omega_d \cos(\omega_d t + \theta) = \sin(\omega_d t + \theta).\zeta\omega_n.\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}$$

$$\Rightarrow \quad \omega_d \cos(\omega_d t + \theta) = \sin(\omega_d t + \theta).\zeta\omega_n$$

$$\Rightarrow \quad \tan(\omega_d t + \theta) = \frac{\omega_d}{\zeta\omega_n}$$

$$\Rightarrow \quad \tan(\omega_d t + \theta) = \frac{\sqrt{1-\zeta^2}}{\zeta}$$

$$\Rightarrow \quad \tan(\omega_d t + \theta) = \tan(\theta)$$

$$\tan(n\pi + \theta) = \tan(\theta)$$

$$\omega_d t = n\pi$$

$$\Rightarrow \quad t = \frac{n\pi}{\omega_d} \quad ;n = 1,2,3, \ldots\ldots$$

$$\boxed{t_p = \frac{\pi}{\omega_d}}$$

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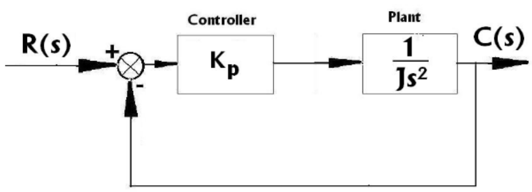
4. Consider the unity feedback system below fig. Let Kp=20 and J=50. Determine the equation of response for a unit step input and determine the steady-state error using the P controller.



Solution

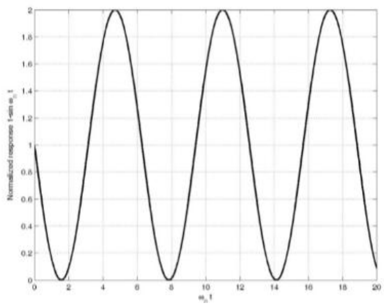
$$\frac{C(s)}{R(s)} = \frac{\frac{K_p}{Js^2}}{1+\frac{K_p}{Js^2}} = \frac{K_p}{Js^2+K_p}$$

$$\frac{C(s)}{R(s)} = \frac{K_p}{J\left(s^2+\omega_n^2\right)}$$

$$\omega_n = \sqrt{\frac{K_p}{J}} = \sqrt{\frac{2}{5}}\ \text{rad/s}$$

$$c(t) = \frac{K_p}{J\omega_n^2}(1-\cos\omega_n t)$$

$$c(t) = \left(1-\cos\sqrt{\frac{2}{5}}t\right)$$



steady-state error

$$s_e(t) = 1-\left(1-\cos\sqrt{\frac{2}{5}}t\right) = \cos\sqrt{\frac{2}{5}}t$$

5. Examine stability of the following system given by

$$s^4+5s^3+2s^2+3s+1=0$$

using Routh-Hurwitz stability criterion. Find the number of roots in the right half of the s-plane and check the stability of the system.

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Solution:

In this problem, Routh's array is

$$
\begin{array}{c|ccc}
s^4 & 1 & 2 & 2 \\
s^3 & 5 & 3 & 0 \\
s^2 & 1.4 & 2 & \\
s^1 & -4.14 & 0 & \\
s^0 & 2 & &
\end{array}
$$

There are two sign changes in first column elements of this array. Therefore, the system is unstable. There are two poles in the right half of the s-plane.