VIT
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

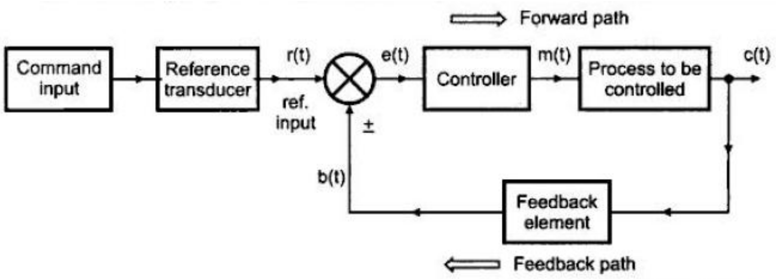
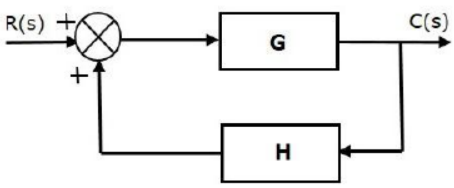
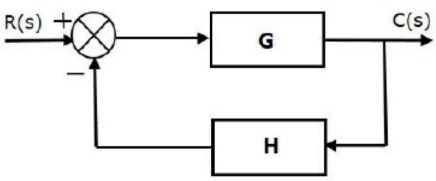
SCHOOL OF MECHANICAL ENGINEERING
CONTINUOUS ASSESSMENT TEST - I
WINTER SEMESTER 2024-2025

SLOT: C2+TC2

Programme Name & Branch : B. Tech – BMA/BME/BMM
Course Code and Course Name : BMEE330L - Control Systems

General instruction(s):

- Answer All Questions
- M - Max mark; CO – Course Outcome; BL – Blooms Taxonomy Level (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create)
- Course Outcomes
  1. Apply the concepts of control systems and modelling techniques.
  2. Develop various representations of system based on the first principles approach.
  3. Infer the domain specifications from the time and frequency response

Q. No	Question	M	CO	BL
1.	Explain the concept of a closed-loop control system with a block diagram, and discuss the role of key elements (reference input, controller, process, sensor, feedback). Also, explain how positive and negative feedback affects system performance, stability, and accuracy.   $T = \frac{G}{1-GH}$  $T = \frac{G}{1+GH}$	10	1	2

Effect of Feedback on Sensitivity

Sensitivity of the overall gain of negative feedback closed loop control system (**T**) to the variation in open loop gain (**G**) is defined as

$$S_G^T = \frac{\frac{\partial T}{T}}{\frac{\partial G}{G}} = \frac{Percentage\ change\ in\ T}{Percentage\ change\ in\ G}$$

Where, **∂T** is the incremental change in T due to incremental change in G.

We can rewrite Equation 3 as

$$S_G^T = \frac{\partial T}{\partial G}\frac{G}{T}$$

Do partial differentiation with respect to G on both sides of Equation 2.

$$\frac{\partial T}{\partial G} = \frac{\partial}{\partial G}\left(\frac{G}{1+GH}\right) = \frac{(1+GH).1-G(H)}{(1+GH)^2} = \frac{1}{(1+GH)^2}$$

From Equation 2, you will get

$$\frac{G}{T} = 1 + GH$$

Substitute Equation 5 and Equation 6 in Equation 4.

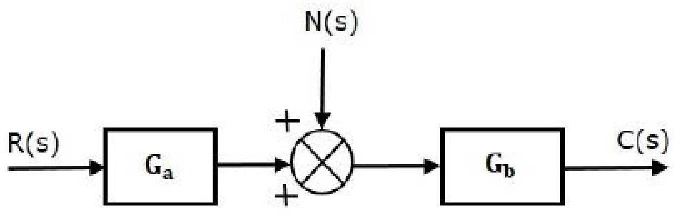
$$S_G^T = \frac{1}{(1+GH)^2}(1+GH) = \frac{1}{1+GH}$$

So, we got the **sensitivity** of the overall gain of closed loop control system as the reciprocal of (1+GH). So, Sensitivity may increase or decrease depending on the value of (1+GH).

Effect of Feedback on Noise

To know the effect of feedback on noise, let us compare the transfer function relations with and without feedback due to noise signal alone.

Consider an **open loop control system** with noise signal as shown below.

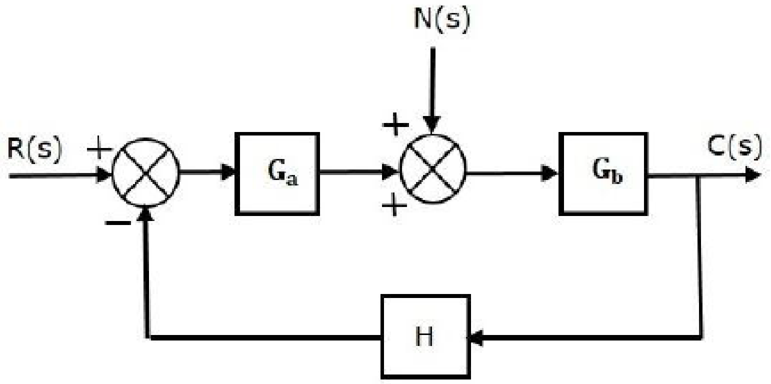


The **open loop transfer function** due to noise signal alone is

$$\frac{C(s)}{N(s)} = G_b$$

It is obtained by making the other input $R(s)$ equal to zero.

Consider a **closed loop control system** with noise signal as shown below.

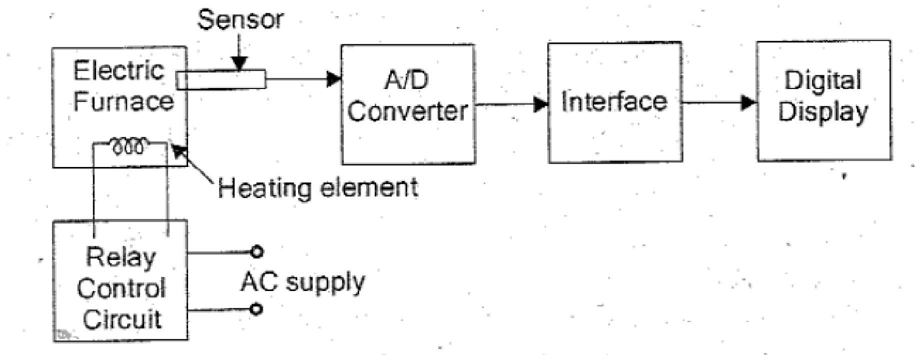
<table>
<tr><td></td><td>


The **closed loop transfer function** due to noise signal alone is

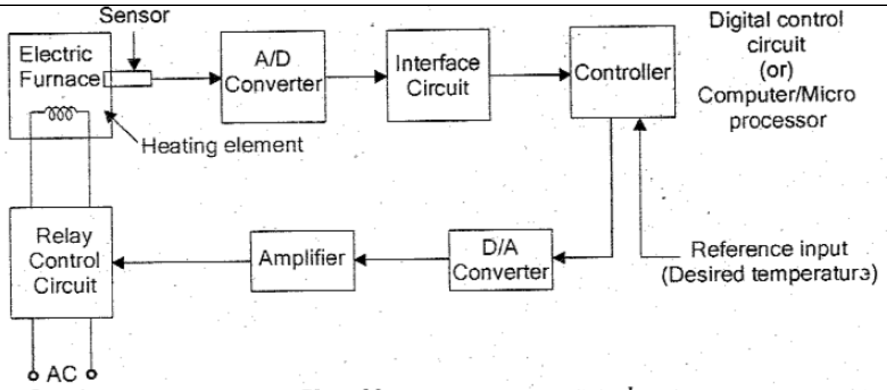
$$\frac{C(s)}{N(s)} = \frac{G_b}{1+G_a G_b H}$$

It is obtained by making the other input $R(s)$ equal to zero.

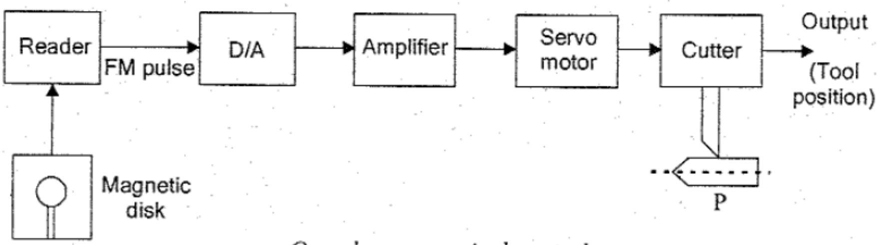
In the closed loop control system, the gain due to noise signal is decreased by a factor of $(1 + G_a G_b H)$ provided that the term $(1 + G_a G_b H)$ is greater than one.
</td><td></td><td></td><td></td></tr>
<tr><td>2.</td><td>
Discuss examples of control systems in various applications, highlighting whether they are open-loop or closed-loop, and explain their role in each. Include at least two examples.



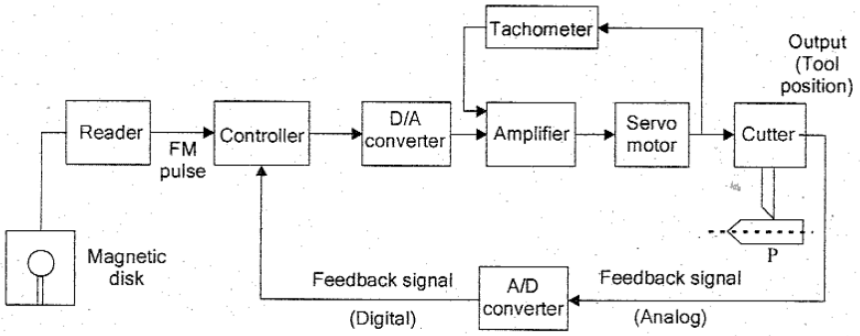
Open loop temperature control system.
</td><td>10</td><td>1</td><td>2</td></tr>
</table>

<table>
<tr><td></td><td>


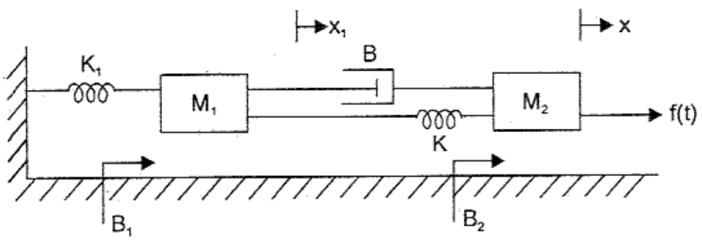
Closed loop temperature control system.



Open loop numerical control system.



Closed loop numerical control system.
</td><td></td><td></td><td></td></tr>
<tr><td>3.</td><td>
Write the differential equations governing the mechanical system shown in fig. and determine the transfer function.



</td><td>10</td><td>2</td><td>4</td></tr>
</table>

Let the displacement of mass M_1 be x_1. The free body diagram of mass M_1 is shown in fig 2. The opposing forces acting on mass M_1 are marked as f_{m1}, f_{b1}, f_b, f_{k1} and f_k.

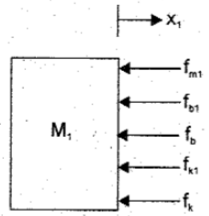
$$f_{m1} = M_1 \frac{d^2x_1}{dt^2}; \qquad f_{b1} = B_1 \frac{dx_1}{dt}; \qquad f_{k1} = K_1 x_1;$$

$$f_b = B\frac{d}{dt}(x_1 - x); \quad f_k = K(x_1 - x)$$

By Newton's second law,

$$f_{m1} + f_{b1} + f_b + f_{k1} + f_k = 0$$

$$\therefore M_1 \frac{d^2x_1}{dt^2} + B_1\frac{dx_1}{dt} + B\frac{d}{dt}(x_1 - x) + K_1x_1 + K(x_1 - x) = 0$$



Free body diagram of mass M_1 (node 1).

On taking Laplace transform of above equation with zero initial conditions we get,

$$M_1s^2X_1(s) + B_1sX_1(s) + Bs\,[X_1(s) - X(s)] + K_1X_1(s) + K\,[X_1(s) - X(s)] = 0$$

$$X_1(s)\,[M_1s^2 + (B_1 + B)s + (K_1 + K)] - X(s)\,[Bs + K] = 0$$

$$X_1(s)\,[M_1s^2 + (B_1 + B)s + (K_1 + K)] = X(s)\,[Bs + K]$$

$$\therefore X_1(s) = X(s)\,\frac{Bs + K}{M_1s^2 + (B_1 + B)\,s + (K_1 + K)}$$

The free body diagram of mass M_2 is shown in fig . The opposing forces acting on M_2 are marked as f_{m2}, f_{b2}, f_b and f_k.

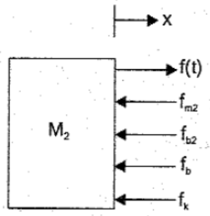
$$f_{m2} = M_2\frac{d^2x}{dt^2} \quad ; \quad f_{b2} = B_2\frac{dx}{dt}$$

$$f_b = B\frac{d}{dt}(x - x_1) \quad ; \quad f_k = K(x - x_1)$$

By Newton's second law,

$$f_{m2} + f_{b2} + f_b + f_k = f(t)$$

$$M_2\frac{d^2x}{dt^2} + B_2\frac{dx}{dt} + B\frac{d}{dt}(x - x_1) + K(x - x_1) = f(t)$$



Free body diagram of mass M_2 (node 2).

On taking Laplace transform of above equation with zero initial conditions we get,

$$M_2s^2X(s) + B_2sX(s) + Bs[X(s) - X_1(s)] + K[X(s) - X_1(s)] = F(s)$$

$$X(s)\,[M_2s^2 + (B_2 + B)s + K] - X_1(s)[Bs + K] = F(s)$$

Substituting for $X_1(s)$ from equation (1) in equation (2) we get,

$$X(s)\,[M_2s^2 + (B_2 + B)s + K] - X(s)\,\frac{(Bs + K)^2}{M_1s^2 + (B_1 + B)s + (K_1 + K)} = F(s)$$

$$X(s)\left[\frac{[M_2s^2 + (B_2 + B)s + K]\,[M_1s^2 + (B_1 + B)s + (K_1 + K)] - (Bs + K)^2}{M_1s^2 + (B_1 + B)\,s + (K_1 + K)}\right] = F(s)$$

$$\therefore \frac{X(s)}{F(s)} = \frac{M_1s^2 + (B_1 + B)s + (K_1 + K)}{[M_1s^2 + (B_1 + B)s + (K_1 + K)]\,[M_2s^2 + (B_2 + B)s + K] - (Bs + K)^2}$$

RESULT

The differential equations governing the system are,

1. $$M_1\frac{d^2x_1}{dt^2}+B_1\frac{dx_1}{dt}+B\frac{d}{dt}(x_1-x)+K_1x_1+K(x_1-x)=0$$

2. $$M_2\frac{d^2x}{dt^2}+B_2\frac{dx}{dt}+B\frac{d}{dt}(x-x_1)+K(x-x_1)=f(t)$$

The transfer function of the system is,

$$\frac{X(s)}{F(s)}=\frac{M_1s^2+(B_1+B)\,s+(K_1+K)}{\left[M_1s^2+(B_1+B)\,s+(K_1+K)\right]\left[M_2s^2+(B_2+B)\,s+K\right]-(Bs+K)^2}$$

4. Derive the governing equations for a simple heat transfer system, considering thermal resistance and capacitance. Explain the assumptions made and the steps involved in formulating the mathematical model.

10 | 2 | 3

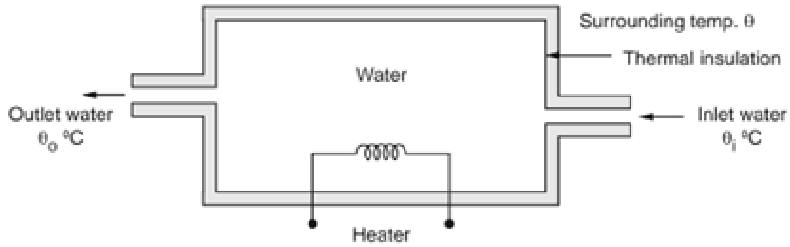
A thermal system used for heating flow of water is shown below.

Electric heating element is provided in the tank to heat the water. The tank is insulated to reduce heat to the surroundings.

The necessary simplifying assumptions are :

1) There is no heat storage in the insulation.

2) All the water in the tank is perfectly mixed and hence at a uniform temperature.



Thermal system

θ_i = Inlet water temperature in °C, θ_o = Outlet water temperature in °C.

θ = Surrounding temperature

q = Rate of heat flow from heating element in J/sec

q_i = Rate of heat flow to the water.

q_t = Rate of heat flow through tank insulation.

C = Thermal capacity in J/°C, R = Resistance of thermal insulation.

So rate of heat flow for the water in tank is, $q_i = C\frac{d\theta_o}{dt}$

The rate of heat flow from the water to the surrounding atmosphere through insulation is,

$$q_t = \frac{\theta_o - \theta}{R}$$

As per the heat transfer principles,

$$q = q_i + q_t$$

Substituting equation

$$q = C\frac{d\theta_o}{dt} + \frac{\theta_o - \theta}{R}$$

Neglecting the term θ/R from the equation (4.10.4) this is because the variation of water temperature θ_o is over and above ambient temperature θ_ω.

$$\therefore \quad q = C\frac{d\theta_o}{dt} + \frac{\theta_o}{R}$$

Taking Laplace transform , $Q(s) = C\,s\,\theta_o(s) + \frac{\theta_o(s)}{R}$

$\therefore$ Transfer function is

$$\boxed{\frac{\theta_o(s)}{Q(s)} = \frac{R}{1 + sCR}}$$

... RC is time constant of the system.

i) Thermal resistance : In the heat transfer system, the thermal resistance is defined as the ratio of change in temperature between the two substances to the change in heat flow rate.

$$\boxed{R = \text{Thermal resistance} = \frac{\text{Change in temperature } ^\circ C}{\text{Change in heat flow rate k cal/sec}}}$$

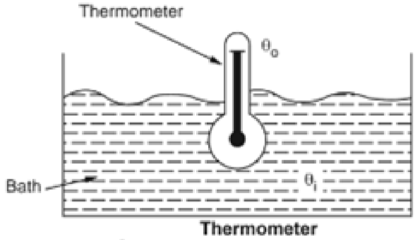
It may be expressed in $^\circ C / J / \min$.

ii) Thermal capacitance : The thermal capacitance is nothing but thermal capacity of the thermal system. It is the ratio of rate of heat storage to the rate of rise of temperature of the tank.

$$\boxed{C = \text{Thermal capacitance} = \frac{\text{Rate of heat storage}}{\text{Rate of rise of temperature}}}$$

It is the product of mass in kg and the specific heat of the liquid in $J/kg\,^\circ C$ hence the thermal capacitance is measured in $J/^\circ C$.

Thermometer



Thermometer

Consider a thermometer placed in a water bath having temperature θ_i, as shown in Fig. 4.10.2.

θ_o is the temperature indicated by the thermometer. The rate of heat flow into the thermometer through its wall is,

$$\frac{dq}{dt} = \frac{\theta_i - \theta_o}{R}, \text{ where R = Thermal resistance of the thermometer wall}$$

The indicated temperature, rises at a rate of,

$$\frac{d\theta_o}{dt} = \frac{1}{C}\frac{dq}{dt}, \text{ where C is thermal capacity of the thermometer.}$$

$$\therefore \quad \frac{d\theta_o}{dt} = \frac{1}{C}\cdot\left[\frac{\theta_i - \theta_o}{R}\right]$$

Taking Laplace of the equation, $s\theta_o(s) = \frac{1}{RC}[\theta_i(s) - \theta_o(s)]$

$$\therefore \quad \boxed{\frac{\theta_o(s)}{\theta_i(s)} = \frac{1}{1+sRC}}$$

... The time constant is RC.

5. The transfer function of a system is given by $(s) = \frac{K(S+6)}{s(S+2)(s+5)(s^2+7s+12)}$.
Determine: (i) Poles (ii) Zeros (iii) Characteristic equation (iv) Pole-zero plot in the s-plane (v) System stability condition.

Marks	CO	BL
10	3	5

Sol. :

i) **Poles are the roots of the equation obtained by equating denominator to zero i.e. roots of**

$$s(s+2)(s+5)(s^2+7s+12) = 0$$

i.e. $s(s + 2)(s + 5)(s + 3)(s + 4) = 0$

So there are 5 poles located at

$s = 0, -2, -5, -3$ and -4

ii) Zeros are the roots of the equation obtained by equating numerator to zero i.e. roots of

$$K(s + 6) = 0$$

i.e. $\quad s = -6$

There is only one zero.

iii) Characteristic equation is one, whose roots are the poles of the transfer function. So it is

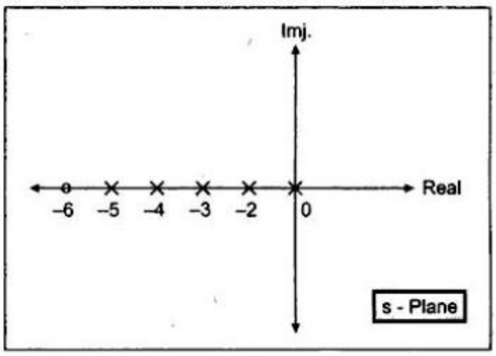
$$s(s + 2)(s + 5)(s^2 + 7s + 12) = 0$$

i.e. $s(s^2 + 7s + 10)(s^2 + 7s + 12) = 0$

i.e $s^5 + 14s^4 + 71s^3 + 154s^2 + 120s = 0$

iv) Pole-zero plot

This is shown in the Fig.



*****