VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF ADVANCED SCIENCES

Fall Semester 2024-2025

Continuous Assessment Test – I

Programme Name & Branch : B.Tech. (Computer Science and Engineering)
Slot: A2+TA2
Course Name & code: Discrete Mathematics and Graph Theory (BMAT205L)
Class Number (s): VL2024250102591, VL2024250102592, VL2024250102590
Faculty Name (s) : Dr. Pallavi Mishra, Dr. Anjaneyulu G.S.G.N, Dr. Gayatri S Panicker
Exam Duration: 90 Min. **Maximum Marks:** 50

General instruction(s): Answer all the questions

Q.No.	Question	Max Marks	CO
1.	a) Write the converse, inverse, and contrapositive of the statement "I go to the college whenever my friends come". Which of these is logically equivalent to the given statement? b) Without using truth table, find PCNF and PDNF for $(p\wedge q)\vee(\sim p\wedge r)$	5 5	CO1
Ans	a) Converse : My friends come whenever I go to the college Inverse : I do not go to the college whenever my friends do not come/ If my friends do not come, I do not go to the college Contrapositive : My friends do not come whenever I do not go to the college/ If I do not go to the college, my friends do not come Contrapositive is logically equivalent to the given statement b) $(p\wedge q)\vee(\sim p\wedge r)$ $\equiv$ ([illegible]) $\equiv (p\vee \sim p)\wedge(p\vee r)\wedge(q\vee \sim p)\wedge(q\vee r)$ $\equiv T\wedge(p\vee r)\wedge(q\vee\sim p)\wedge(q\vee r)$ $\equiv (p\vee r\vee F)\wedge(q\vee\sim p\vee F)\wedge(q\vee r\vee F)$ $\equiv [p\vee r\vee(q\wedge\sim q)]\wedge[q\vee\sim p\vee(r\wedge\sim r)]\wedge[q\vee r\vee(p\wedge\sim p)]$ $\equiv (p\vee r\vee q)\wedge(p\vee r\vee\sim q)\wedge(q\vee\sim p\vee r)\wedge(q\vee\sim p\vee\sim r)\wedge(q\vee r\vee p)\wedge(q\vee r\vee\sim p)$ $\equiv (p\vee q\vee r)\wedge(p\vee\sim q\vee r)\wedge(\sim p\vee q\vee r)\wedge(\sim p\vee q\vee\sim r)$ Required PCNF. PDNF : $(\sim p\wedge\sim q\wedge r)\vee(\sim p\wedge q\wedge r)\vee(p\wedge q\wedge\sim r)\vee(p\wedge q\wedge r)$		
2.	Using Indirect method of proof, show that the premises "My father praises me only if I can be proud of myself. Either I do	10	CO1

<table>
<tr><td></td><td>well in sports or I can not be proud of myself. If I study hard, then I cannot do well in sports” lead to the conclusion “If father praises me, then I do not study well".</td><td></td><td></td></tr>
<tr><td>Ans</td><td>
p: My father praises me
q: I am proud of myself
r: I do well in sports
s: I study hard

Premises: $p \to q$
$r \vee \sim q$
$s \to \sim r$
Conclusion: $p \to \sim s$

31 Friday
Indirect method!
Include $\sim(p \to \sim s) \equiv \sim(\sim p \vee \sim s) \equiv p \wedge s$ as Additional premice
<table>
<tr><td>Steps</td><td>Premises</td><td>Rules</td></tr>
<tr><td>1</td><td>$r \vee \sim q$</td><td>Rule P</td></tr>
<tr><td>2.</td><td>$s \to \sim r$</td><td>Rule P</td></tr>
<tr><td>3.</td><td>$\sim s \vee \sim r$</td><td>Rule T; $p \to q \equiv \sim p \vee q$</td></tr>
<tr><td>4.</td><td>$\sim r \vee \sim s$</td><td>$p \vee q \equiv q \vee p$</td></tr>
<tr><td>5.</td><td>$\sim q \vee \sim s$</td><td>Rule T, Resolution {1,4}</td></tr>
<tr><td>6.</td><td>$q \to \sim s$</td><td>$p \to q \equiv \sim p \vee q$</td></tr>
<tr><td>2023 7.</td><td>$p \to q$</td><td>Rule P</td></tr>
<tr><td>8</td><td>$p \to \sim s$</td><td>Rule T, H.S {6,7} Saturday 1</td></tr>
<tr><td>9</td><td>$p \wedge s$</td><td>A.P.</td></tr>
<tr><td>10</td><td>p</td><td>Simplification {9}</td></tr>
<tr><td>11</td><td>$\sim s$</td><td>M.P. {8,10}</td></tr>
<tr><td>12</td><td>s</td><td>Simplification {9}</td></tr>
<tr><td>13.</td><td>$s \wedge \sim s \equiv F$
Contradiction</td><td>Conjunction {11,12}</td></tr>
</table>
</td><td></td><td></td></tr>
<tr><td>3.</td><td>Verify the validity of the following argument
“A student has not completed his daily homework. Everyone in the class completed their monthly assignments. Therefore, someone who has completed his monthly assignments has not completed his daily homework”</td><td>10</td><td>CO1</td></tr>
<tr><td>Ans</td><td>C(x) : x is in this class (or) x is a student in this class
B(x) : x has completed daily homework
P(x) : x has completed the monthly assignments
$\exists x(C(x) \wedge \neg B(x))$
$\forall x(C(x) \to P(x))$
$\therefore\ \exists x(P(x) \wedge \neg B(x))$</td><td></td><td></td></tr>
</table>

	Valid Argument: **Step** — **Reason** 1. $\exists x(C(x) \land \neg B(x))$ — Premise 2. $C(a) \land \neg B(a)$ — EI from (1) 3. $C(a)$ — Simplification from (2) 4. $\forall x(C(x) \to P(x))$ — Premise 5. $C(a) \to P(a)$ — UI from (4) 6. $P(a)$ — MP from (3) and (5) 7. $\neg B(a)$ — Simplification from (2) 8. $P(a) \land \neg B(a)$ — Conj from (6) and (7) 9. $\exists x(P(x) \land \neg B(x))$ — EG from (8)		
4.	Verify whether $(Z^+, *)$, where * is defined as $a * b = a + b + 1$ (+ is the ordinary addition) forms a group, where Z^+ is the set of positive integers.	10	CO2
Ans	Not a group as identity does not exist.		
5.	Prove that the order of a subgroup of a finite group divides the order of the group.	10	CO2
Ans	Proof of Lagrange's Theorem		