



VIT[®]

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

REG. NO.:

SLOT: C2+TC2+TCC2

SCHOOL OF ADVANCED SCIENCES
DEPARTMENT OF MATHEMATICS
FALL SEMESTER – 2024-2025
Continuous Assessment Test – II

Programme Name & Branch : B.Tech
 Course Code & Course Name : BMAT205L and Discrete Mathematics and Graph Theory
 Class Number(s) : 2538,2559,2562,2563,2564,2565, 2567,2569,2570,2571
 2572,2573,2576,2579,2581,2583,2586
 “Common Slot QP”
 Faculty Name(s) : Prof. EZHILMARAN D, PRAVEEN, VIJAYAKUMAR V,
 NIRANJAN HARI, DEEPA G, JAYALAKSHMI M, SUMIT,
 AMRIT DAS, PARTHIBAN A, VENKATESWARLU B,
 MURUGAN V, NALLIAH M, MANIMARAN A, JUHI BAROI,
 ANKUSH CHANDA, SARAVANARAJAN M C, JAGATHESWARI
 Date of Examination and Time : 15-10-2024 and 9:30AM -11:00AM
 Exam Duration : 90 Minutes Maximum Marks : 50

General Instruction(s):

- Examination mode is an Open Text Book / Open Note Book. Students are permitted to bring any number of textbooks, printouts of e-books (complete / chapters) and handwritten notebooks (class notes).
- M-Max mark; CO-Course Outcome; BL-Blooms Taxonomy Level (1 -Remember, 2-Understand, 3-Apply, 4-Analyse, 5-Evaluate, 6-Create)
- **Answer ALL Questions.**
- Course Outcomes-CO2:Use algebraic structures in applications,
CO3:Counting techniques in engineering problems
CO4:Use lattice and Boolean algebra properties in Digital circuits.

Q. No.	Questions	M	CO	BL
1.	An encoding function $e : B^3 \rightarrow B^7$ is given by the generator matrix G is $\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$ <p>(i). Determine all the code words (ii). Find the associated parity-check matrix (iii). Use the parity-check matrix to decode the received words 1000111 and 1100110.</p>	10	2	3

Q. No.	Questions	M	CO	BL
2.a	A copying company has eight photocopying machines and seven employees who can operate them. There are four copying jobs to be done. How many ways are there to allocate these jobs to operators and machines? Give detail answer.	5	3	3
2.b	How many different words of five letters can you make from the letters of the word REPUBLICAN, if every word must contain two different vowels and three different consonants? Give detail answer.	5	3	3
3.	Use generating function to solve the recurrence relation $a_k = 5a_{k-1} - 6a_{k-2}$ with initial conditions $a_0 = 6$ and $a_1 = 30$.	10	3	5
4.	Let (L, \leq) be the poset, where $L = \{1, 2, 3, 6, 12\}$ and the relation " \leq " means "divides". (i). Draw the Hasse diagram for the poset (L, \leq) (ii). Find the least upper bound and greatest lower bound tables for every two elements of the poset (L, \leq) . Hence, prove that the poset (L, \leq) is lattice.	10	4	2
5.	Let B be the set of all positive integer divisors of 15, that is, $B = \{1, 3, 5, 15\}$. Prove that B forms a Boolean algebra with zero element 1 and unity element 15, provided the operations are defined as follows: (i). $x + y$ is the least common multiple of x and y , (ii). $x \times y$ is the greatest common divisor, and (iii). x' is the ordinary arithmetical quotient $\frac{15}{x}$.	10	4	2



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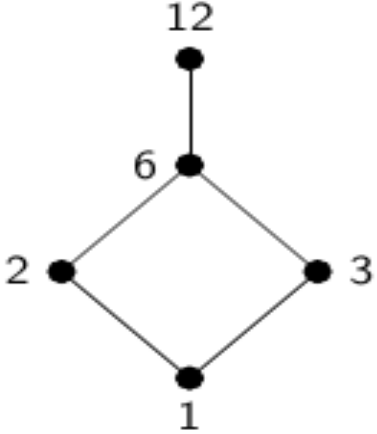
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FALL SEMESTER – 2024-2025
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Programme Name & Branch : B.Tech
Course Code & Course Name : BMAT205L and Discrete Mathematics and Graph Theory
Class Number(s) : “Common Slot QP”
Faculty Name(s) : “Common Slot QP”
Date of Examination and Time : 15-10-2024 and 9:30AM -11:00AM
Exam Duration : 90 Minutes Maximum Marks : 50

Q. No.	Questions	M	CO	BL
1.	<p>For(i) Let $e : B^3 \rightarrow B^7$ be an encoding function. Then $B^3 = \{000, 010, 011, 001, 100, 101, 110, 111\}$. The generator matrix $G = (I_3 A)$ is given as</p> $G = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$ <p>Then the code words $e(w) = wG$, where $w \in B^3$. Therefore, we get</p> $\begin{aligned} e(000) &= [000] \times G = [0000000] \\ e(010) &= [010] \times G = [0101011] \\ e(011) &= [011] \times G = [0111100] \\ e(001) &= [001] \times G = [0010011] \\ e(100) &= [100] \times G = [1001001] \\ e(101) &= [101] \times G = [1011010] \\ e(110) &= [110] \times G = [1100110] \\ e(111) &= [111] \times G = [1110100] \end{aligned}$ <p>For(ii). the parity check matrix $H = (A^T I_4)$ is,</p> $H = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$ <p>For (iii), If r is a received word then decode of r is $H \times r^T$. So, If $r = [1000111]$ then $H \times [1000111]^T = [1010]$ If $r = [1100110]$ then $H \times [1100110]^T = [0000]$.</p>			

Q. No.	Questions	M	CO	BL
2.a	Call the jobs A,B,C,D. Choose two arrangements: first, which four operators should do the jobs; second, which machines should be used. The operator choice can be made in $P(7, 4)$ ways, and the machines in $P(8, 4)$ ways. In each case, the first member of the sequence is the one allocated to job A, the second to job B, and so on. There are $P(7, 4) \times P(8, 4) = 76548765 = 1, 411, 200$ ways.	5	3	3
2.b	The three consonants can be chosen in $C(6, 3) = 20$ ways, and the vowels in $C(4, 2) = 6$ ways. After the choice is made, the letters can be arranged in $5! = 120$ ways. So there are $20 \times 6 \times 120 = 14, 400$ "words."	5	3	3
3.	<p>$a_k = 5a_{k-1} - 6a_{k-2}$ with initial conditions $a_0 = 6$ and $a_1 = 30$. The generating function is</p> $G(x) = \sum_{k=0}^{\infty} a_k x^k = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots + \dots$ <p>Now, we take</p> $\sum_{k=2}^{\infty} a_k x^k = 5 \sum_{k=2}^{\infty} a_{k-1} x^k - 6 \sum_{k=2}^{\infty} a_{k-2} x^k \quad (1)$ $\begin{aligned} \sum_{k=2}^{\infty} a_k x^k &= a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \dots \\ &= a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + \dots - a_0 x^0 - a_1 x^1 \\ &= G(x) - a_0 x^0 - a_1 x^1 \end{aligned}$ $\begin{aligned} \sum_{k=2}^{\infty} a_{k-1} x^k &= a_1 x^2 + a_2 x^3 + a_3 x^4 + a_4 x^5 + \dots \\ &= x[a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + \dots] - a_0 x \\ &= xG(x) - a_0 x \end{aligned}$ $\begin{aligned} \sum_{k=2}^{\infty} a_{k-2} x^k &= a_0 x^2 + a_1 x^3 + a_2 x^4 + a_3 x^5 + \dots \\ &= x^2[a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + \dots] \\ &= x^2 G(x) \end{aligned}$ <p>substituting the above in the equation (1), we get</p> $\begin{aligned} G(x) - a_0 - a_1 x &= 5x(G(x) - a_0) - 6x^2 G(x) \\ 6x^2 G(x) - 5xG(x) + G(x) &= a_0 + a_1 x - 5xa_0 \\ G(x)[6x^2 - 5x + 1] &= a_0 + a_1 x - 5xa_0 \\ G(x) &= \frac{a_0 + a_1 x - 5xa_0}{6x^2 - 5x + 1} \\ G(x) &= \frac{6 + 30x - 5x(6)}{6x^2 - 5x + 1} \\ G(x) &= \frac{6}{6x^2 - 5x + 1} \text{ (by partial fraction)} \\ G(x) &= \frac{-12}{1 - 2x} + \frac{18}{1 - 3x} \end{aligned}$ <p>According the formula $\frac{1}{1-ax} = \sum_{n=0}^{\infty} a^n x^n$, we get</p>	10	4	2

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3.	$G(x) = \sum_{k=0}^{\infty} (-12)2^k x^k + \sum_{k=0}^{\infty} 18(3)^k x^k$ $= \sum_{k=0}^{\infty} [18(3)^k - 12(2^k)]x^k$ <p>Hence $a_k = 18(3)^k - 12(2^k), k = 0, 1, 2, 3, 4...$</p>																																							
4.	<p>For (i)</p>  <p>For(ii) $\text{lub}_{a,b} = \text{LCM}_{a,b}$ (LCM means least common multiple). Least upper bound(lub) table</p> <table border="1" data-bbox="220 1153 1082 1621"> <tr> <td>lub</td> <td>1</td> <td>2</td> <td>3</td> <td>6</td> <td>12</td> </tr> <tr> <td>1</td> <td>1</td> <td>2</td> <td>3</td> <td>6</td> <td>12</td> </tr> <tr> <td>2</td> <td>2</td> <td>2</td> <td>6</td> <td>6</td> <td>12</td> </tr> <tr> <td>3</td> <td>3</td> <td>6</td> <td>3</td> <td>6</td> <td>12</td> </tr> <tr> <td>6</td> <td>6</td> <td>6</td> <td>6</td> <td>6</td> <td>12</td> </tr> <tr> <td>12</td> <td>12</td> <td>12</td> <td>12</td> <td>6</td> <td>12</td> </tr> </table>	lub	1	2	3	6	12	1	1	2	3	6	12	2	2	2	6	6	12	3	3	6	3	6	12	6	6	6	6	6	12	12	12	12	12	6	12	10	4	2
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	<p>glb_{a,b}=GCD_{a,b} (GCD means greast common divisor). Greatest lower bound(glb) table</p> <table border="1" data-bbox="220 255 1082 725"> <tr> <td>glb</td> <td>1</td> <td>2</td> <td>3</td> <td>6</td> <td>12</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> </tr> <tr> <td>2</td> <td>1</td> <td>2</td> <td>1</td> <td>1</td> <td>2</td> </tr> <tr> <td>3</td> <td>1</td> <td>1</td> <td>3</td> <td>3</td> <td>3</td> </tr> <tr> <td>6</td> <td>1</td> <td>2</td> <td>3</td> <td>6</td> <td>6</td> </tr> <tr> <td>12</td> <td>1</td> <td>2</td> <td>3</td> <td>6</td> <td>12</td> </tr> </table> <p>clearly from the above tables, glb and lub exist for every two elements. Hence It is lattice.</p>					glb	1	2	3	6	12	1	1	1	1	1	1	2	1	2	1	1	2	3	1	1	3	3	3	6	1	2	3	6	6	12	1	2	3	6	12																	
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