VIT
Vellore Institute of Technology

Final Assessment Test – May 2024

Course: MAT1014 - Discrete Mathematics and Graph Theory
Class NBR(s): 1762 / 1764 / 1766 / 1772 Slot: C1+TC1+TCC1+V2
Time: Three Hours Max. Marks: 100

- KEEPING MOBILE PHONE/ELECTRONIC DEVICES EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- DON'T WRITE ANYTHING ON THE QUESTION PAPER

Answer any TEN Questions
(10 X 10 = 100 Marks)

1. Find the principal conjunctive normal form and principal disjunctive normal for the statement, $(P \to (Q \wedge R)) \wedge (\neg P \to (\neg Q \wedge \neg R))$. **[10]**

2. Show that the following set of premises is inconsistent. **[10]**
   If Jack gets his degree, then he will go for a job.
   If Jack goes for a job, then he will get good salary.
   If he goes for higher study, then he will not get good salary.
   Jack gets his degree and goes for higher study.

3. Show by indirect method of proof, that **[10]**
   $\forall x(p(x) \vee q(x)) \Rightarrow (\forall x\, p(x)) \vee (\exists x\, q(x))$.

4. State and prove Lagrange's theorem. **[10]**

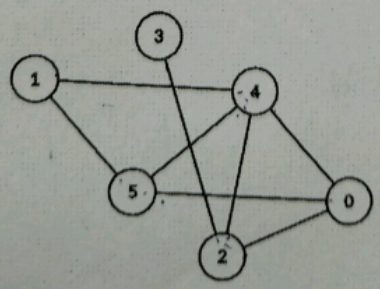
5. Show that the set Q^+ of all positive rational numbers forms an abelian group under the operation $*$ defined by $a * b = \frac{1}{2}(ab); \forall a, b \in Q^+$. **[10]**

6. Draw the Hasse diagram for the Poset, **[10]**
   $(\{1\},\{2\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,3,4\},\{2,3,4\},\subseteq)$ and find the following:
   (i) Minimal elements and Least element.
   (ii) Maximal elements and Greatest element.
   (iii) Least Upper Bound of $\{\{2\},\{4\}\}$.
   (iv) Greatest Lower Bound of $\{\{1,3,4\},\{2,3,4\}\}$.

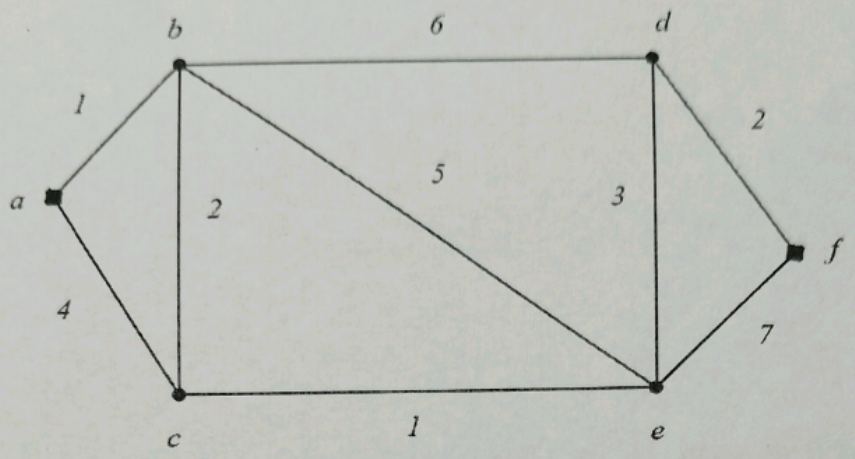
7. Let $(L, \leq)$ be a lattice in which $*, \oplus$ denote the operations of meet and join respectively. For any $a, b \in L$, prove that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$. **[10]**

8. Minimize the following Boolean expression using Karnaugh Map. **[10]**
   $f(a,b,c,d) = \Sigma(0,2,3,7,11,13,14,15)$.

9. a) Show that the number of vertices of odd degree in an undirected graph is always even. **[10]**
   b) Write the adjacency matrix for the following graph.

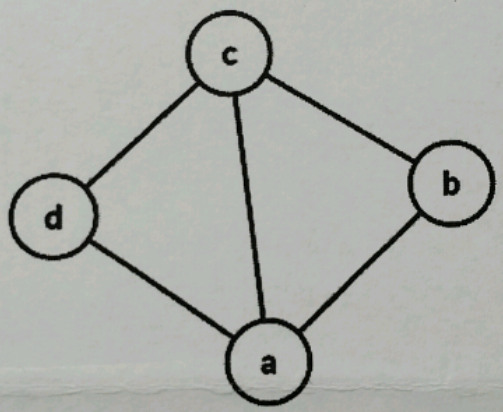


10. Apply the Dijisktra's Algorithm to the weighted graph given below and find the shortest path between the vertices a *to* f and hence compute the total weight of the shortest path between the mentioned vertices. **[10]**



11. Show that a tree with *n* vertices has (*n*-1) edges. **[10]**

12. Find the chromatic polynomial of the following graph. **[10]**



⇔⇔⇔ M/E/TX⇔⇔⇔