BI/K/TX

Reg. No:

VIT
Vellore Institute of Technology

Final Assessment Test – November 2024

Course: BMAT203L - Linear Algebra and Differential Equations

Class NBR(s): 2604 Slot: C1+TC1+TCC1

Time: Three Hours Max. Marks: 100

- KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- DON'T WRITE ANYTHING ON THE QUESTION PAPER

Answer ALL Questions

(10 X 10 = 100 Marks)

1. Consider the following system of linear equations [10]

$$x + 2y - z + 2\alpha\, w = 1$$
$$x + y + 2z - \alpha\, w = 2$$
$$x + 2y - 2z + \alpha^2\, w = \beta.$$

(i) Reduce the augmented matrix of the given system into its row echelon form.

(ii) Find the conditions on α, and β such that the system will have (I) a unique solution (II) infinitely many solutions (III) no solution.

2. Find the eigenvalues and eigenvectors of the matrix [10]

$$A = \begin{pmatrix} 4 & 0 & 1 \\ -1 & -6 & -2 \\ 5 & 0 & 0 \end{pmatrix}.$$

3. (i) Check whether the sequence $a_n = \sqrt{n^2 + 4n} - n$ is decreasing or increasing. [5]

(ii) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{6^n}$. If the series is convergent, find the sum of the series. [5]

4. Examine the convergence or the divergence of the series [10]

$$\frac{x}{1.3} + \frac{x^2}{3.5} + \frac{x^3}{5.7} + \cdots$$

5. Find the third degree Taylor polynomial of the function $f(x) = e^{2x}\cos(3x)$ in power of x. [10]

6. Suppose a student carrying a flu virus returns to an isolated college campus of 1000 students. If it is determined that the rate at which the virus spreads is proportional not only to the number P(t) of students infected but also to the number of students not infected. Determine the number of infected students after 6 days given that the number of infected students after 4 days is 50. [10]

7. Solve the differential equation $t\frac{dy}{dt} + y = (t^2 + 1)e^t, y(0) = 1.$ [10]

8. Consider the following predator-prey model [10]

$\frac{dx}{dt} = x - 0.03\, xy;\ \frac{dy}{dt} = -0.4y + 0.01\, xy$, with $x(0) = 15, y(0) = 15.$

Find $x(1), y(1)$ using Euler's method. Assume $\Delta t = \frac{1}{3}$.

9.a) Solve the following system of differential equations [10]

$\frac{dy}{dt} = x^2 - 4y;\ \frac{dx}{dt} = 3x$, with the initial conditions $y(0) = 1,\ x(0) = 3$.

OR

9.b) Considered the mass-spring system modelled by the differential equation [10]

$y''(t) + 4y(t) = \cos(t)$, with the initial conditions $y(0) = 1,\ y'(0) = 1$. Find velocity at any time t.

10.a) Formulate the governing equation of the damped harmonic oscillator with mass $m = 1$, spring constant $k = 2$, and the damping coefficient $c = 3$. Find the oscillator position at any time t, when the initial displacement is zero and initial velocity is 1. Also, classify the oscillator. [10]

$m\frac{d^2y}{dt} + 1V + \frac{3}{2}$

OR

10.b) Solve the system of linear differential equations [10]

$\frac{dx}{dt} = 4x + 5y;\ \frac{dy}{dt} = -5x - 4y$, with $x(0) = 1, y(0) = 0$.

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