


Final Assessment Test - May 2024

 Course: **BMAT202L - Probability and Statistics**

 Class NBR(s): 1661 / 1664 / 1669 / 1670 / 1672 / 1745 /
 2268 / 2282 / 2294 / 2299

 Slot: **D1+TD1**

 Time: **Three Hours**

 Max. Marks: **100**

- **KEEPING MOBILE PHONE/ELECTRONIC DEVICES EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE**
- **DON'T WRITE ANYTHING ON THE QUESTION PAPER**

General Instructions:

Statistical Tables are permitted

 Answer any **TEN** Questions
 (10 X 10 = 100 Marks)

1. The following data represent the lifetimes (in hours) of a sample of 40 transistors: [10]

112, 121, 126, 108, 141, 104, 136, 134

121, 118, 143, 116, 108, 122, 127, 140

113, 117, 126, 130, 134, 120, 131, 133

118, 125, 151, 147, 137, 140, 132, 119

110, 124, 132, 152, 135, 130, 136, 128

Determine the sample mean, median and mode.

2. Find the mean and standard deviation of the following frequency distribution and [10]
 hence obtain the value of coefficient of variation:

Class	500-700	700-900	900-1100	1100-1300	1300-1500
Frequency	5	11	26	10	8

3. A random variable X has the probability density function: [10]

$$f(x) = \begin{cases} 2x; & 0 < x < 1 \\ 0; & \text{otherwise} \end{cases}$$

 Find (i) $P\left(X < \frac{1}{2}\right)$, (ii) $P\left(\frac{1}{4} < X < \frac{1}{2}\right)$ (iii) $P\left(X > \frac{3}{4} \mid X > \frac{1}{2}\right)$ and
 iv) $P\left(X < \frac{3}{4} \mid X > \frac{1}{2}\right)$.

4. Three balls are drawn at random without replacement from a box containing [10]
 2 white, 3 red and 4 black balls. If X denotes the number of white balls drawn and
 Y denotes the number of red balls drawn find the joint probability distribution of
 X, Y . Also, compute its mean.



5. A group of twelve children participated in a psychological study designed to assess the relationship if any, between age, x years, and average total sleep time (ATST), y minutes. To obtain a measure for ATST, recordings were taken on each child on five consecutive nights and then averaged. The results obtained are shown in the table: [10]

Child	Age (x years)	ATST (y minutes)
A	4.4	586
B	6.7	565
C	10.5	515
D	9.6	532
E	12.4	478
F	5.5	560
G	11.1	493
H	8.6	533
I	14.0	575
J	10.1	490
K	7.2	530
L	7.9	515

Calculate the value of the correlation coefficient between x and y and comment on your values.

A student counted the number of words in an essay she had written, recording the total every 10 lines. [10]

z-test

No. of lines (x)	10	20	30	40	50	60	70	80
No. of words (y)	75	136	210	291	368	441	519	588

- (i) Find the equation or a line to convert lines to words. How many words (approximately) has she written if she writes
 - (a) 65 lines (b) 100 lines (c) 1000 lines?
- (ii) Are you happy with all these estimates?

7. A Council is considering whether to base a recovery vehicle on a stretch of road to help clear incidents as quickly as possible. The road concerned carries over 5000 vehicles during the peak rush hour period. Records show that, on average, the number of incidents during the morning rush hour is 5. The Council won't base a vehicle on the road if the probability of having more than 5 incidents in any one morning is less than 30%. Based on this information should the Council provide a vehicle? [10]

Poisson

8. The mean yield for one-acre plot is 662 kilos with a standard deviation 32 kilos. Assuming normal distribution, how many one-acre plots in a batch of 1,000 plots would you expect to have yield (i) over 700 kilos (ii) below 650 kilos and (iii) between 650 kilos and 700 kilos. [10]

9. It is hoped that a newly developed pain reliever will more quickly produce perceptible reduction in pain to patients after minor surgeries than a standard pain reliever. The standard pain reliever is known to bring relief in an average of 3.5 minutes with standard deviation 2.1 minutes. To test whether the new pain reliever works more quickly than the standard one, 50 patients with minor surgeries were given the new pain reliever and their times to relief were recorded. The experiment yielded sample mean $\bar{x} = 3.1$ minutes and sample standard deviations $s = 1.5$ minutes. Is there sufficient evidence in the sample to indicate, at the 5% level of significance, that the newly developed pain reliever does deliver perceptible relief more quickly? [10]

10. A random sample of 10 hot drinks from dispenser A had a mean volume of 203 ml and a standard deviation, s , of 3 ml. A random sample of 15 hot drinks from dispenser B gave corresponding values of 206 ml and 5 ml. The amount dispensed by each machine may be assumed to be normally distributed. Test at the 5% significance level, the hypothesis that there is no difference in the variability of the volume dispensed by the two machines. [10]

11. In a comparison of the cleaning action of four detergents, 20 pieces of white cloth were first soiled with India ink. The cloths were then washed under controlled conditions with 5 pieces washed by each of the detergents. Unfortunately three pieces of cloth were 'lost' in the course of the experiment. Whiteness readings, made on the 17 remaining pieces of cloth, are shown below: [10]

Detergent			
A	B	C	D
77	74	73	76
81	66	78	85
61	58	57	77
76	-	69	64
69	-	63	-

Assuming all whiteness readings to be normally distributed with common variance, test the hypothesis of no difference between the four brands as regards mean whiteness readings after washing.

12. The density function of the time failure in years of the gizmos manufactured by a certain company is given by [10]

$$f(t) = \frac{200}{(t+10)^3}, t \geq 0.$$

- (i) Derive the reliability function and determine the reliability for the first year of operation.
- (ii) Compute the MTTF.
- (iii) What is the design life for a reliability 0.95?
- (iv) Will a one-year burn-in period improve the reliability in part (i)? If so, what is the new reliability?

⇔⇔⇔ J/E/TX ⇔⇔⇔