Final Assessment Test – May 2024

VIT
Vellore Institute of Technology

Course: BECE207L - Random Processes
Class NBR(s): 1279 / 1284 / 1287 / 1292 / 1295 / 1298 / 5616
Slot: E1+TE1
Time: Three Hours
Max. Marks: 100

- KEEPING MOBILE PHONE/ELECTRONIC DEVICES EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- DON'T WRITE ANYTHING ON THE QUESTION PAPER

Answer ALL Questions
(10 X 10 = 100 Marks)

1. The joint PDF of random variables (X, Y) is

$$f_{X,Y}(x,y) = \frac{1}{4}e^{-|x|-|y|}, -\infty < x \leq \infty, -\infty < y \leq \infty$$

(a) Are X and Y statistically independent random variables?
(b) Calculate the probability for $X \leq 1$ and $Y \leq 0$.

2. Determine the PDF of $X = U_1 + U_2$, where $U_1 \sim U(0,1), U_2 \sim U(0,1)$ and U_1, U_2 are independent. Where U represents Uniform Random Variable.

3.(a) The joint density function of X and Y is given by

$$f_{XY}(x,y) = \frac{1}{y}e^{-\left(y+\frac{x}{y}\right)} x > 0, y > 0$$

Find E[X], E[Y] and show that Cov (X, Y) = 1

OR

3.(b) The random variables X and Y are independent Gaussian with zero mean and equal variance σ^2. If Z and θ are another two random variables defined as $Z = \sqrt{X^2 + Y^2}$ and $\theta = \tan^{-1}\frac{Y}{X}$, $|\theta| < \pi$. Find the joint density function of Z and θ.

4. A random process $X(t) = B\cos(50t + \phi)$ where B and ϕ are independent random variables. B is a random variable with mean '0' and variance '1' and ϕ is uniformly distributed in interval $(-\pi, \pi)$. Find the mean and auto correlation of process $X(t)$.

5. a) Statistically independent zero mean random process X(t) and Y(t) have autocorrelation function $R_{XX}(\tau) = e^{-|\tau|}$ and $R_{YY}(\tau) = \cos 2\pi\tau$ respectively. Find the autocorrelation function of the sum $Z(t) = X(t) + Y(t)$.

b) A radioactive source emits particles at a rate of 5 per minute in accordance with Poisson process. Find the probability that 10 particles are recorded in 4 min period.

6. Consider a random process $Y(t) = X(t+a) - X(t-a)$, where $X(t)$ is a WSS random process with autocorrelation $R_{xx}(\tau)$. Prove that

$$R_{YY}(\tau) = 2R_{XX}(\tau) - R_{XX}(\tau + 2a) - R_{XX}(\tau - 2a)$$

Also, find the PSD $S_{YY}(\omega)$ in terms of $S_{XX}(\omega)$.

7.(a) Two jointly stationary random processes $X(t)$ and $Y(t)$ have the cross-spectral density.

$$S_{XY}(\omega) = \frac{6}{(j\omega)^2 + 7j\omega + 10}$$

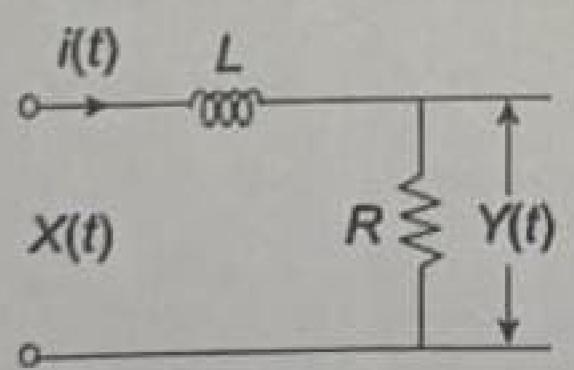
Determine the corresponding cross-correlation function.

OR

7.(b) A zero mean white noise with noise power density $\frac{N_0}{2}$ is applied to a filter with transfer function $H(\omega) = \frac{1}{1+j\omega}$

i) What is the average power of the output
ii) Find $S_{YY}(\omega)$ and $R_{YY}(\tau)$.
iii) Find $S_{YX}(\omega)$ and $R_{YX}(\tau)$.

8. The input voltage to an RL, Low pass filter circuit is a stationary random process $X(t)$ with $E[X(t)] = 2$ and $R_{XX}(\tau) = 4 + e^{-2|\tau|}$. Let $Y(t)$ be the voltage across the resistor. Find the power spectral density of output process $Y(t)$.



9. White noise with power density $\frac{N_o}{2}$ is applied to a low pass network for which $|H(0)| = 2$. It has a noise bandwidth of 2MHz. If the average output noise power is 0.1 W in a 1 ohm resistor, what is N_o?

10. A signal $x(t) = u(t)5t^2\exp(-2t)$, where u(t) is unit step signal is added to white noise for which $N_0/2 = 10^{-2}$ W/Hz. The sum is applied to a matched filter.

(a) What is the filter's transfer function?
(b) What is (S_o/N_o) ?
(c) Sketch the impulse response of the filter.
(d) Is the filter realizable?

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