SLOT: F2 + TF2

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**VIT** Vellore Institute of Technology (Deemed to be University under section 3 of UGC Act, 1956)

**SCHOOL OF MECHANICAL ENGINEERING**
**CONTINUOUS ASSESSMENT TEST (CAT)-I - KEY**
**WINTER SEMESTER 2024-2025**

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|---|---|
| **Programme Name & Branch** | **: B.Tech & BME, BMA, BMM** |
| **Course Code** | **: BMEE402L** |
| **Course Name** | **: Heat and Mass Transfer** |
| **Faculty Name(s)** | **: TAPANO KUMAR HOTTA, CHIRANJEEVI C, KAMATCHI R, DEEPAKKUMAR R, ASHISH ALEX SAM** |

| Q. No | Question | M |
|---|---|---|
| 1. | Apply one dimensional Laplace equation of cylindrical co-ordinate system and derive the temperature distribution and heat loss for constant surface boundary conditions. | 10 |
| ANS | | |

Winter 2024-25 Heat & Mass Transfer (1)

CAT-1-KEY

(1) 1-D Laplace equation for Cylindrical co-ordinates

$$\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\cdot\frac{\partial T}{\partial r}\right)=0 \quad \text{— (1M)}$$

Integrate the differential equation.

$$\int r\cdot\frac{\partial T}{\partial r}=\int c_1 \Rightarrow \frac{\partial T}{\partial r}=\frac{c_1}{r}.$$

$$\Rightarrow \boxed{T = C_1\cdot\ln r + C_2} \quad (2M).$$

Boundary conditions:

at $r=r_i$; $T=T_1$ and at $r=r_o$; $T=T_2$

subst. above boundary condition in the temp. distribution Equation to find $C_1$ & $C_2$.

we'll get.

$$\boxed{T = \left[\frac{T_1-T_2}{\ln\left(\frac{r_i}{r_o}\right)}\right]\ln r + \left[T_1-\frac{(T_1-T_2)}{\ln\left(\frac{r_i}{r_o}\right)}\right]\times\ln r_i} \quad (4M)$$

Heat flow/Heat loss is:

$$Q=-K\cdot A\cdot\frac{dT}{dr} \quad (\text{or}) \quad Q=-K\cdot A_i\cdot\left.\frac{dT}{dr}\right|_{r=r_i}$$

$$\Rightarrow Q=-K\cdot(2\pi \cancel{r_i} L)\left[\frac{1}{\cancel{r_i}}\times\frac{(T_1-T_2)}{\ln\left(\frac{r_i}{r_o}\right)}\right]$$

$$\boxed{Q=\frac{(T_1-T_2)}{\left[\frac{\ln(r_o/r_i)}{2\pi kL}\right]}} \quad (3M).$$

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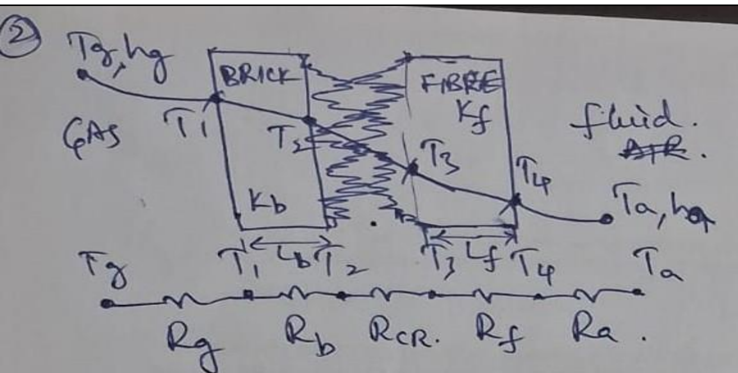
| | | |
|---|---|---|
| 2. | A composite wall is made of brick and fibre materials. The thermal conductivity and thickness of brick and fibre are 0.4 W/m.K, 0.05 W/m.K and 15 cm and 10 cm respectively. The thermal contact resistance at the interface between the brick and fibre is 0.002 $m^2$.K/W. The brick material is exposed to flue gases at 250 °C with a convective heat transfer coefficient of 15 $W/m^2$.K and the fibre material is surrounded with a fluid at 50 °C with a convective heat transfer coefficient of 22 $W/m^2$.K. Estimate the rate of heat transfer through the composite wall per unit area and also the overall heat transfer coefficient. | 10 |
| ANS | (see below) | |

(2)




$k_b = 0.4\ W/mK$.
$k_f = 0.05\ W/mK$.
$L_b = 0.15\ m$
$L_f = 0.1\ m$
$R_{CR} = 0.002\ m^2K/W$
$T_g = 250°C$
$h_g = 15\ W/m^2K$.
$T_a = 50°C$
$h_a = 22\ W/m^2K$.

$Q = ?$.
$U = ?$.

Assume area, $A = 1\ m^2$

$R_b = \frac{L_b}{K_b \cdot A} = \frac{0.15}{0.4 \times 1} = 0.375\ K/W$.

$R_f = \frac{L_f}{K_f \cdot A} = \frac{0.1}{0.05 \times 1} = 2\ K/W$.

$R_{CR} = \frac{0.002}{1} = 0.002\ K/W$.

$R_g = \frac{1}{h_g \cdot A} = \frac{1}{15 \times 1} = 0.0666\ K/W$.

$R_a = \frac{1}{h_a \cdot A} = \frac{1}{22 \times 1} = 0.04545\ K/W$.

(5M).

Total Resistance,

$\boxed{\Sigma R = R_g + R_b + R_{CR} + R_f + R_a = 2.48905\ K/W}$ (1M).

Rate of Heat Transfer, $Q = \frac{(T_g - T_a)}{\Sigma R} = \frac{(250 - 50)}{2.48905}$

$\boxed{Q = 80.352\ W}$ (2M)

Overall Heat Transfer Co-efficient, U:

$Q = U \cdot A \cdot (T_g - T_a) \Rightarrow U = \frac{Q}{A \cdot (T_g - T_a)} = \boxed{0.40176\ W/m^2K} = U$.

(2M).

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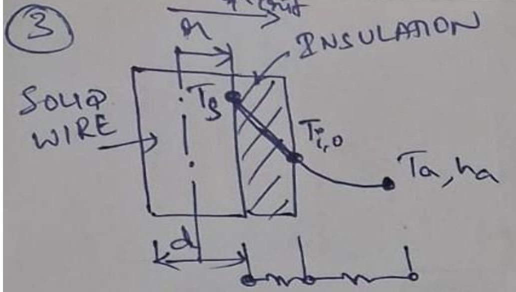
**WINTER SEMESTER 2024-2025**

| | | |
|---|---|---|
| 3. | A solid wire of 6.5 mm diameter surface is maintained at a uniform temperature of 60 °C. It is covered with an insulating material having thermal conductivity of 0.174 W/m.K, which is exposed to atmospheric air at 20 °C having a convection heat transfer co efficient is 9 $W/m^2.K$. Determine the thickness of insulation for maximum heat loss, and heat loss per meter length? Also find the percentage increase in the heat dissipation with insulation compared to without insulation. | 10 |

ANS

(3) (3)




$d = 6.5\times10^{-3}$ m

$r = 3.25\times10^{-3}$ m.

$T_s = 60°C$

$K_{ins} = 0.174$ W/mK.

$T_a = 20°C$ ; $h_a = 9$ W/m²K.

find: ✓ Thickness of insulation for max. heat transfer, $t$ = ?

✓ $Q$ = ? W/m        $t$ = ? m

✓ % increase in heat loss?

(1) Maximum heat loss is at critical radius ($r_c$)

$\therefore r_c = \frac{K_{ins}}{h_a} = \frac{0.174}{9} = 0.01933$ m (2M)

(for a cylinder)

(2) Thickness of insulation, $t = r_c - r$ =

$t = 0.01933 - 0.00325 = 0.01608$ m (1M)

(3) Heat loss per meter length WITHOUT insulation:

$Q_{WITHOUT} = h_a \cdot A \cdot (T_s - T_a) = 9\times\pi\times0.0065\times(60-20)$ (2M)

$= 7.35$ W/m

(4) Heat loss per meter length WITH insulation:

$$Q_{WITH} = \frac{(T_s - T_a)}{\left(\left[\frac{\ln(r_c/r)}{2\pi K_{in} L}\right] + \left[\frac{1}{h_a \cdot 2\pi r_c L}\right]\right)} = 15.71 \text{ W/m}$$ (3M)

(5) % increase in heat dissipation with insulation: (2M)

$$= \left(\frac{Q_{WITH} - Q_{WITHOUT}}{Q_{WITHOUT}}\right)\times100 = \left(\frac{15.71 - 7.35}{7.35}\right)\times100 = 113.74\%$$

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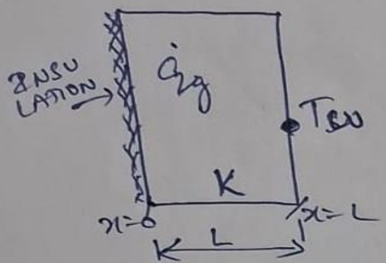
| | | |
|---|---|---|
| 4. | A plane wall of 0.5 m thick is insulated on one side while the other side surface is maintained at a temperature of 300 °C. The wall is made with a material of thermal conductivity 30 W/m.K and it is generating a uniform heat of 400 W/m$^3$. Assuming steady one dimensional heat transfer, find (a) the temperature at the mid plane, (b) the maximum temperature in the wall and the location of the plane where it occurs. | 10 |

ANS

4.




$L = 0.5\,m$.

$T_w = 300°C$

~~$K = 300\,W/mK$~~ $K = 30\,W/mK$.

$\dot{q}_g = 400\,W/m^3$.

(a) $T_{at\,x=L/2} = ?$

(b) $T_{max} = ?$ ; (c) $x = ?$ @ $T_{max}$.

1-D Steady state heat conduction with heat source for plane slab.

differential Eq. is: $\frac{d^2T}{dx^2} + \frac{\dot{q}_g}{K} = 0$.

1st Integrating above Eq. $\Rightarrow \boxed{\frac{dT}{dx} = -\frac{\dot{q}_g}{K}\cdot x + C_1}$ — (1)

2nd integration gives as $\Rightarrow \boxed{T_x = -\frac{\dot{q}_g}{K}\cdot\frac{x^2}{2} + C_1\cdot x + C_2}$ — (2).

Boundary conditions are: at $x=0$; $\frac{dT}{dx} = 0$ (BC 1) and at $x=L$ ~~$T=300$~~ $T = T_w$ (BC 2)

Eq. (1) & B.C (1) $\Rightarrow C_1 = 0$ — (3).

Eq. (2), (3) & B.C (2) $\Rightarrow T_w = -\frac{\dot{q}_g}{K}\cdot\frac{L^2}{2} + 0 + C_2 \Rightarrow C_2 = T_w + \frac{\dot{q}_g L^2}{2K}$.

Now, subst. $C_1$ & $C_2$ in Eq. (2)

$T = \Rightarrow \boxed{T = -\frac{\dot{q}_g}{2K}\cdot x^2 + T_w + \frac{\dot{q}_g}{2K}\cdot L^2}$ (5M).

(a) $T_{at\,x=L/2} \Rightarrow T = -\frac{400}{2\times 30}\times(0.25)^2 + 300 + \frac{400}{2\times 30}\times(0.5)^2$ (2M)

$= 301.25\,°C$ → (1M)

(c) $T_{max}$ is at $\frac{dT}{dx} = 0$. i.e. at $x = 0$.

(b) $T_{max} = -\frac{\dot{q}_g}{2K}\cdot(0)^2 + T_w + \frac{\dot{q}_g}{2K}\cdot L^2 = 300 + \frac{400}{2\times 30}\times(0.5)^2 = 301.67°C$ (2M)

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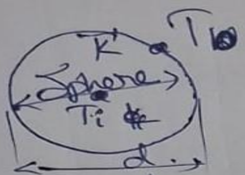
| | | |
|---|---|---|
| 5. | Consider a homogeneous spherical piece of radioactive material of radius $r_o = 0.04$ m that is generating heat at a constant rate of $q_{gen} = 4 \times 10^7$ W/m$^3$. The heat generated is dissipated to the environment steadily. The outer surface of the sphere is maintained at a uniform temperature of 80 °C and the thermal conductivity of the sphere is 15 W/m.K. Assuming one dimensional heat transfer, (a) express the differential equation and the boundary conditions for heat conduction through the sphere, (b) determine the temperature at the centre of the sphere. | 10 |

ANS

(5).




Radioactive material

$d = 0.08$ m

$R_o = 0.04$ m

$\dot{q}_g = 4 \times 10^7$ W/m$^3$.

$T_\infty = 80°C$

$K = 15$ W/mK.

(a) differential Eq for the given system is:

$$\frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \cdot \frac{\partial T}{\partial r}\right) + \frac{\dot{q}_g}{K} = 0.$$

Boundary Conditions:

at $r = 0$ ; $\frac{dT}{dr} = 0$.

$r = R_o$ ; $T = T_\infty$ i.e at $r = 0.04$ ; $T = 80°C$

(5M)

(b) Temperature at the center of sphere; $(T_i)$

From Data Book:

$$T_i = T_o + \frac{\dot{q}_g}{6K} R_o^2$$

$$= 80 + \frac{4 \times 10^7}{6 \times 15} \times (4 \times 10^{-2})^2$$

$$T_i = 791.11°C$$

(5M)

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