7. Let $T: R^3 \to R^3$ be a linear transformation such that [10]

$$T(1,0,0) = (2,-1,4)$$
$$T(0,1,0) = (1,5,-2)$$
$$T(0,0,1) = (0,3,1)$$

Find the formula for $T(x,y,z)$ and compute $T(2,3,-2)$.

8. Let $B_1 = \{(-6,4),(8,-4)\}$ and $B_2 = \{(-2,4),(4,-4)\}$ be basis for R^2 and let $A = \begin{bmatrix} -4 & 14 \\ -6 & 14 \end{bmatrix}$ be the matrix relative to basis B_1 in $T: R^2 \to R^2$. Find the similar matrix A' relative to basis B_2. [10]

9. Apply the Gram-Schmidt process to transform the basis vectors $u_1 = (2,2,2), u_2 = (0,2,2), u_3 = (0,0,2)$ into an orthogonal basis $\{v_1, v_2, v_3\}$, and then normalize the orthogonal basis vectors to obtain an orthonormal basis $\{q_1, q_2, q_3\}$. [10]

10.a) Find the interpolating polynomial for the given four points $(0,6),(2,0),(-2,4),(6,12)$. [10]

OR

10.b) The approximate average temperature of a city for the months {January, May, August, December} are given by the following table: Fit the cubic polynomial satisfying the data. [10]

Month	1	5	8	12
Avg. temp.	3	24	31	5

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Reg. No:

Final Assessment Test – November 2024

VIT
Vellore Institute of Technology

Course: UMAT201L - Linear Algebra

Class NBR(s): 2593 / 2595 / 2597 / 2600

Slot: B1+TB1

Time: Three Hours

Max. Marks: 100

- KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- DON'T WRITE ANYTHING ON THE QUESTION PAPER

Answer ALL Questions

(10 X 10 = 100 Marks)

1. Using Gauss-Jordan elimination method find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 1 & 0 & 2 \end{bmatrix}$. [10]

2.a) Solve the system of linear equations $Ax = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 4 & 1 & 0 & 1 \\ -2 & 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 7 \end{bmatrix} = \mathrm{b}$ [10]

by using an LU factorization of A.

OR

2.b) Solve the given system of linear equations using LU-factorization method. [10]

$$2x_1 + 2x_2 + 2x_3 = 2,$$
$$8x_1 + 6x_2 - x_3 = 12,$$
$$6x_1 + 10x_2 + 6x_3 = 8.$$

3. Consider the set of all 2×2 diagonal matrices. Determine if this set, along with the standard matrix operations of addition and scalar multiplication, forms a vector space. If it does not, identify at least one of the vector space axioms that is violated. [10]

4. Determine whether the set $S = \{(1,2,3), (1,4,9), (1,8,27)\}$ forms a basis for $\mathbb{R}^3$. Provide a justification for your answer. [10]

5. Let the matrix A is [10]

$$A = \begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & -1 & -3 & 1 & 3 \\ -2 & -1 & 1 & -1 & 3 \\ 0 & 3 & 9 & 0 & -12 \end{bmatrix}$$

i. Find the rank, basis for null space, and nullity of A.
ii. Find a subset of the column vectors of A that forms a basis for the column space of A.

6. Consider the linear transformation $T: R^3 \to R^3$ defined by [10]

$$T(x_1, x_2, x_3) = (2x_1 + 3x_2 + x_3, 3x_1 + 3x_2 + x_3, 2x_1 + 4x_2 + x_3).$$

i. Show that T is invertible, and find its inverse.
ii. Find the Ker(T).
iii. Find the Range (T).
iv. Find the basis for Range (T).