BH/K/TX

Reg. No:

Final Assessment Test – November 2024

VIT
Vellore Institute of Technology

Course: UMAT201L - Linear Algebra

Class NBR(s): 2594 / 2596 / 2598

Time: Three Hours

Slot: B2+TB2

Max. Marks: 100

- KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- DON'T WRITE ANYTHING ON THE QUESTION PAPER

Answer ALL Questions

(10 X 10 = 100 Marks)

1. Use $X = A^{-1} \cdot B$ to solve the given system of linear equations, where X denote unknown matrix, A is coefficient matrix and B is constant matrix. [10]

$$x - 2y + 3z = 9,$$
$$-x + 3y = -4,$$
$$2x - 5y + 5z = 17.$$

2. Solve the given system of linear equations using LU-factorization method. [10]

$$x_1 + x_2 + x_3 = 1,$$
$$4x_1 + 3x_2 - x_3 = 6$$
$$3x_1 + 5x_2 + 3x_3 = 9$$

3. Verify the set of all real numbers $(x, y) \in \mathrm{R}^2$ where $x > 0$ under the given operations is vector space or not? [10]

$$(x_1, y_1) + (x_2, y_2) = (x_1 \cdot x_2, y_1 + y_2);$$
$$k(x, y) = (x^k, ky), k \in R$$

If it is not a vector space, then list out all the axioms which are failed with the examples.

4. a) Determine the given polynomials are linear independent or linearly dependent in P_2. [10]

$$p_1 = 2 - 2x, p_2 = 10 + 6x - 4x^2, p_3 = 2 + 6x - 2x^2.$$

b) Verify the set W is a subspace of R^3.

$$W = \{(2t, 3t, 4t) \in R^3 : t \in R\}.$$

5.a) Find Row-space, column-space and Null space of the given matrix A, and hence verify Rank-Nullity theorem. [10]

$$A = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & 4 & 2 & -5 & -4 \end{bmatrix}$$

OR

5.b) Let V and W be the two subspaces of R^4 with bases [10]

$V = \{(x_1, x_2, x_3, x_4) \in R^4 : x_2 + x_3 + x_4 = 0\}$,

$W = (x_1, x_2, x_3, x_4) \in R^4 : x_1 + x_2 = 0, x_3 = 2x_4$.

Find basis and dimension of $V, W, V + W, V \cap W$.

6. Consider the linear transformation $T: R^3 \rightarrow R^3$ defined by $T(x, y, z) = (2x + 3y + z, 3x + 3y + z, 2x + 4y + z)$. Determine the invertibility of T, if T is invertible then, find $T^{-1}(x, y, z)$. [10]

7. Let $T: R^3 \rightarrow R^2$ be the transformation. Find the formula for $T(x, y, z)$ and the basis are given by $\alpha = \{(1,1,1,),(1,1,0),(1,0,0)\}$ and $\beta = \{(1,0),(2,-1),(4,3)\}$. Also for $T: \alpha \rightarrow \beta$, find the linear transformation $T(x, y, z)$ and compute $T(5,4,3)$. [10]

8. Let $B_1 = \{(-3,2),(4,-2)\}$ and $B_2 = \{(-1,2),(2,-2)\}$ be basis for R^2 and let $A = \begin{bmatrix} -2 & 7 \\ -3 & 7 \end{bmatrix}$ be the matrix relative to basis B_1 in $T: R^2 \rightarrow R^2$. Find the similar matrix A' relative to basis B_2. [10]

9. Apply the Gram-Schmidt process to transform the basis vectors $u_1 = (1,1,1), u_2 = (0,1,1), u_3 = (0,0,1)$ into an orthogonal basis $\{v_1, v_2, v_3\}$, and then normalize the orthogonal basis vectors to obtain an orthonormal basis $\{q_1, q_2, q_3\}$. [10]

10.a) Find the polynomial $p(x) = a + bx + cx^2 + dx^3$ that satisfies $p(0) = 2,\ p'(0) = 4,\ p(1) = 8,\ p'(1) = 8$. Here $p'(x) = \frac{dp(x)}{dx}$. Draw the graph to show the interpolation or fitting. [10]

OR

10.b) Decode the encrypted word "**GTNKGKDUSK**" using Hill 2-cipher method and the original message was encrypted using the matrix $\begin{bmatrix} 5 & 6 \\ 2 & 3 \end{bmatrix}$. [10]

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