I/E/TX

VIT

Final Assessment Test – May 2024

Course: BMAT202L - Probability and Statistics

Class NBR(s): 1663 / 1676 / 1678 / 1680 / 1682 / 1684 / 1740 / 1742 / 1746 / 4940

Slot: B2+TB2

Time: Three Hours

Max. Marks: 100

- KEEPING MOBILE PHONE/ELECTRONIC DEVICES EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- DON'T WRITE ANYTHING ON THE QUESTION PAPER

General Instructions :

Statistical tables are permitted

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. The following table showing the wage distribution in a certain factory, Weekly wages [10]

Weekly wages	20-40	40-60	60-80	80-100	100-120	120-140	140-160	160-180	180-200
No. of Employees	8	12	20	30	40	35	18	7	5

Determine the mean, median and mode.

2. Compute standard-deviation and range from the following data: [10]

Height in cms	158	159	160	161	162	163	164	165	166
No. of Persons	15	20	32	35	33	22	20	10	8

3. a) If the random variable X takes the values 1, 2, 3 and 4 such that $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$. Find the probability distribution of X. [5]

b) Consider the density function $f(x) = k\sqrt{x} \quad 0 < x < 1$
$= 0 \quad$ otherwise [5]

Evaluate k and find the cumulative distribution function of $F(x)$. Also find $P\{X < 0.5\}$.

4. Let X denotes the number of times a certain numerical control machine will malfunction: 1, 2 or 3 times on any given day. Let Y denote the number of times a technician is called on an emergency call. Their joint probability distribution is given as [10]

$p(x,y)$		x		
		1	2	3
y	1	0.05	0.05	0.10
	3	0.05	0.10	0.35
	5	0.00	0.20	0.10

(i) Evaluate the marginal distribution of X and Y

(ii) Find $P(Y = y/X = x)$ and

(iii) Find the mean of X and Y.

5. Find out the coefficient of correlation between the sales and expenses of the following 10 firms: [10]

Firms	1	2	3	4	5	6	7	8	9	10
Sales	50	50	55	60	65	65	65	60	60	50
Expenses	11	13	14	16	16	15	15	14	13	13

6. Determine the multiple regression line for the following data of a transport company on the weights of 6 shipments, the distances they were moved and the damage of the goods that was incurred. [10]

Weight X_1(1000kg)	4.0	3.0	1.6	1.2	3.4	4.8
Distance X_2(100km)	1.5	2.2	1.0	2.0	0.8	1.6
DamageY (Rs.)	160	112	69	90	123	18

Estimate the damage when a shipment of 3700 kg is moved to a distance of 260 km.

7. It is conjectured that an impurity exists in 30% of all drinking wells in a certain rural community. In order to gain some insight into the true extent of the problem, it is determined that some testing is necessary. It is too expensive to test all wells in the area, so 10 are randomly selected for testing. Determine the probability that exactly 3 wells have the impurity, assuming that the conjecture is correct? [10]

8. a) On an average 20 red blood cells are found in a fixed volume of blood for a normal person. Determine the probability that the blood sample of a normal person will contain less than 15 red cells. [5]

b) Assume that the 'reduction' of a person's oxygen consumption during a period of Transcendenta Meditation(T.M) is a continuous random variable X, normally distributed with mean 37.6 cc/mt and s.d 4.6 cc/mt. Determine the probability that during a period of T.M. a person's oxygen consumption will be reduced by (i) at least 44.5 cc/mt (ii) at most 35.0 cc/mt (iii) anywhere from 30.0 to 40.0 cc/mt. [5]

9. A vote is to be taken among the residents of a town and the surrounding county to determine whether a proposed chemical plant should be constructed. The construction site is within the town limits, and for this reason many voters in the county believe that the proposal will pass because of the large proportion of town voters who favour the construction. To determine if there is a significant difference in the proportions of town voters and country voters favoring the proposal, a poll is taken. If 120 of 200 town voters favor the proposal and 240 of 500 county residents favor it, would you agree that the proportion of town voters favoring the proposal is higher than the proportion of county voters? Use an $\alpha = 0.05$ level of significance. [10]

10. The following data represent the running times of films produced by two motion-picture companies: [10]

Company	Time(minutes)
1	102 86 98 109 92
2	81 165 97 134 92 87 114

Test the hypothesis that the average running time of films produced by company 2 exceeds the average running time of films produced by company 1 by 10 minutes against the one-sided alternative that the difference is less than 10 minutes. Use a 0.1 level of significance and assume the distributions of times to be approximately normal with unequal variances.

11. Four different machines, M1, M2, M3 and M4, are being considered for the assembling of a particular product. It was decided that six different operators would be used in a randomized block experiment to compare the machines. The machines were assigned in a random order to each operator. The operation of the machines requires physical dexterity and it was anticipated that there would be a difference among the operators in the speed with which they operated the machines. The amounts of time (in seconds) required to assemble the product are shown in table: [10]

Machine	1	2	3	4	5	6	Total
1	42.5	39.3	39.6	39.9	42.9	43.6	247.8
2	39.8	40.1	40.5	42.3	42.5	43.1	248.3
3	40.2	40.5	41.3	43.4	44.9	45.1	255.4
4	41.3	42.2	43.5	44.2	45.9	42.3	259.4
Total	163.8	162.1	164.9	169.8	176.2	174.1	1010.9

Test the null hypothesis, at the 0.05 level of significance, that the machines perform at the same mean rate of speed by ANOVA.

12. a) An electronic circuit consists of 5 silicon transistors, 3 silicon diodes, 10 composition resistors and 2 ceramic capacitors connected in series configuration. The hourly failure rate of each component is given below: [5]

silicon transistor: $\lambda_t = 4 \times 10^{-5}$

silicon diode : $\lambda_d = 3 \times 10^{-5}$

composition resistor: $\lambda_r = 2 \times 10^{-4}$

ceramic capacitor: $\lambda_c = 2 \times 10^{-4}$

Calculate the reliability of the circuit for 10 hours, when the components follow exponential distribution.

b) A cutting tool wears out with a time to failure that is normally distributed. It is known that about 34.5% of the tools fail before 9 working days and about 78.8% fail before 12 working days. (i) Compute the MTTF (ii) Determine its design life for a reliability of 0.99 (iii) Determine the probability that the cutting tool will last one more day given that it has been in use for 5 days. [5]

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