C/L/TY

Reg. No: 25BBT0001

# Final Assessment Test – November 2025

VIT® Vellore Institute of Technology

Course: BAMAT101 - Multivariable Calculus and Differential Equations

Class NBR(s): 5311 / 5317 / 5449 / 5452 / 5456 / 5602 / 5608

Slot: G1+TG1

Time: Three Hours

Max. Marks: 100

- ➤ KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- ➤ DON'T WRITE ANYTHING ON THE QUESTION PAPER

| COs | CO Statements |
|---|---|
| CO1 | To evaluate partial derivatives and solve optimization problems involving several variables with or without constraints. |
| CO2 | To solve multiple integrals in cartesian, polar, cylindrical and spherical coordinates. |
| CO3 | To understand gradient, directional derivatives, divergence, curl, Green's, Stokes and Gauss Divergence theorems. |
| CO4 | To utilize different solution methods to obtain the solution for second-order ordinary differential equations. |
| CO5 | To demonstrate proficiency in solving linear and nonlinear first-order partial differential equations. |

**BL – Blooms Taxonomy Level (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create)**

**Answer ALL Questions**

**(10 X 10 = 100 Marks)**

1. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then show that CO1 BL2

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}.$$

2. A solid metallic sphere of radius 1 m is placed in a region where the temperature at any point $(x, y, z)$ inside or on the sphere is given by $u(x, y, z) = 400\, xyz^2$ (in °C). Engineers want to determine the locations on the surface of the sphere where the temperature is highest, as these points represent potential hot spots that may require cooling or insulation to prevent thermal damage. Find: CO1 BL2

   (I) The maximum temperature on the surface of the sphere $x^2 + y^2 + z^2 = 1$.

   (II) The points on the sphere $(x, y, z)$ where this highest temperature occurs.

3.a) Change the order of integration in CO2 BL3

$$\int_0^1 \int_y^{2-y} xy\, dx\, dy$$

and hence evaluate it.

**OR**

3.b) Transform the integral into polar coordinates and hence evaluate CO2 BL3

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\, dy\, dx .$$

4\. Find the volume cut off from the sphere $x^2+y^2+z^2=a^2$ by the cylinder $x^2+y^2=ax$. CO2 BL2

5\. Find $a$ and $b$ such that the surfaces $ax^2-byz=(a+2)x$ and $4x^2y+z^3=4$ cut orthogonally at $(1,-1,2)$. 5 CO3 BL2

6.a) Verify the Gauss divergence theorem for $\vec{F}=4xz\vec{i}-y^2\vec{j}+yz\vec{k}$ over the cube bounded by $x=0, x=1, y=0, y=1, z=0, z=1$. CO3 BL3

**OR**

6.b) A fluid dynamics engineer is analyzing a two-dimensional flow field over a planar region in the $xy$-plane. The flow is described by the vector field CO3 BL3

$$F(x,y)=(u,v)=(xy+y^2,x^2).$$ 10

Verify Green's theorem by calculating the circulation of the flow around a channel when the engineer considers a closed boundary $C$ formed by the curves $y=x$ and $y=x^2$.

7\. Determine the solutions of a mechanical system with weight 2.45kg, stiffness constant 4 N/m, damping force is 2 times instantaneous velocity and external force is 8 sin $4t$. The initial conditions are $y(0)=1$ and $y'(0)=0$. 5 CO4 BL2

8\. Solve $(x+1)^2D^2y+(x+1)Dy=(2x+3)(2x+4)$. CO4 BL2

9\. Solve $(x^2-yz)p+(y^2-zx)q=z^2-xy$. 6 CO5 BL2

10\. By the method of separation of variables, solve CO5 BL2

$$4\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=3z$$ 10

subject to $z=e^{-5y}$ when $x=0$.

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