REG.NO.:

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF ADVANCED SCIENCES
CONTINUOUS ASSESSMENT TEST - II
FALL SEMESTER 2025-2026

SLOT: B2+TB2

Programme Name & Branch : B.Tech & All the Branches
Course Code and Course Name: BAMAT101, Multivariable Calculus and Differential Equations
Class Number(s) : Common to B2 slot
Date of Examination : 06-10-2025
Exam Duration : 90 minutes **Maximum Marks: 50**

General instruction(s): Answer All Questions Open Text Book / Open Note Book mode

- M - Max mark; CO – Course Outcome; BL – Blooms Taxonomy Level (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create)
- Course Outcomes
- **CO2**. Solve multiple integrals in cartesian, polar, cylindrical and spherical coordinates.
  CO3. Understand gradient, directional derivatives, divergence, curl, Green's, Stokes and Gauss Divergence theorems.
  CO4. Utilize different solution methods to obtain the solution for second-order ordinary differential equations.

Q. No	Question	M	CO	BL
1.	Evaluate the double integral $\iint_R \left(\sqrt{9-x^2-y^2}\right) dA$,where R is the region bounded by the circle $x^2+y^2=9$ by converting the integral into polar co-ordinates.	10	2	2
2.	Evaluate the triple integral $\iiint_V (x^2+y^2) dV$ by converting to cylindrical coordinates, where V is the region given by $V=(x,y,z): x^2+y^2 \le 4; 0 \le z \le 3$.	10	2	3
3.	a) Find the directional derivative of the function $f = xy^2 + yz^2 + zx^2$ along the tangent to the curve $x=t, y=t^2, z=t^3$ at the point (1, 1, 1).	5	3	2
	b) In robotics path planning, a robot is moving in a 3-dimensional environment modeled by the potential field $\vec{F} = (y^2 \cos x + z^3)\vec{i} + (2y\sin x - 4)\vec{j} + 3xz^2\vec{k}$. Show that this field represents a conservative artificial potential field, and compute the scalar potential function that the robot would use for energy-efficient navigation.	5		
4.	a) A virtual reality (*VR*) haptic device applies a force to a user's hand, modeled by the vector field $\vec{F} = (x^2 - y^2 + x)\vec{i} - (2xy + y)\vec{j}$, where *(x,y)* represents the position of the hand in meters in a *2D* plane. If the user moves their hand along the straight line path from point *A(0,0)* to point *B(1,1).* Find the work done by the force along path $y^2 = x$.	5	3	3
	b) Using Green's theorem evaluate, $\int_C (y - \sin x)dx + \cos x\, dy$, where C is the plane triangle bounded by the lines $y=0,\ x=\frac{\pi}{2},\ y=\left(\frac{2}{\pi}\right)x$.	5		
5.	An electric circuit consists of $R = 40\Omega, L = 1H, C = 16\times10^{-4} F$ .The electrical oscillations in the circuit are free. If the initial charge and current are both zero. Determine the charge and current at time.	10	4	3

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Course Code and Course Name : **BAMAT101, Multivariable Calculus and Differential Equations**
Class Number(s) : **Common to B2 slot**
Date of Examination : **06-10-2025**
Exam Duration : **90 minutes** **Maximum Marks: 50**

Q. No	Question
1.	**Evaluate the double integral** $\iint_R \left(\sqrt{9-x^2-y^2}\right)dA$ **,where *R* is the region bounded by the circle** $x^2+y^2=9$ **by converting the integral into polar co-ordinates.** **Solution:** $x=r\cos\theta,\ y=r\sin\theta,\ dA=r\ drd\theta$, $x^2+y^2=9$ **is a circle with centre (0,0) and radius '3'.** **Limits:** $r:0\to 3$, $\theta:0\to\frac{\pi}{2}$, $I=\int_0^{\frac{\pi}{2}}\int_0^3\sqrt{9-r^2}\ \ rdrd\theta,\ 9-r^2=t,\ rdr=-\frac{dt}{2},\ t:9\to 0$ $I=-\frac{1}{2}\int_0^{\frac{\pi}{2}}\int_9^0\sqrt{t}\ \left(-\frac{dt}{2}\right)d\theta=\frac{9\pi}{2}$. **(10Marks)**
2.	**Evaluate the triple integral** $\iiint_V (x^2+y^2)dV$ **by converting to cylindrical coordinates, where *V* is the region given by** $V=(x,y,z):x^2+y^2\le 4;\ 0\le z\le 3$. **Solution:** $x=r\cos\theta,\ y=r\sin\theta,\ z=z,\ dV=r\ drd\theta dz$ **Limits:** $r:0\to 2$, $\theta:0\to 2\pi$, $z:0\to 3$, $I=\int_{\theta=0}^{2\pi}\int_{r=0}^{2}\int_{z=0}^{3} r^3\ dzdrd\theta=24\pi$. **(10Marks)**
3.	**a) Find the directional derivative of the function** $f=xy^2+yz^2+zx^2$ **along the tangent to the curve** $x=t,y=t^2,z=t^3$ **at the point (1, 1, 1).** **Solution:** $(gradf)_{(1,1,\ 1)}=3\vec{i}+3\vec{j}+3\vec{k},\ \left(\frac{\vec{dr}}{dt}\right)_{(1,1,1)}=\vec{i}+2\vec{j}+3\vec{k},\ \frac{\vec{dr}}{dt}$ **is the vector along with the tangent to the curve. Unit vector along with the tangent =** $\hat{e}=\frac{\vec{i}+2\vec{j}+3\vec{k}}{\sqrt{14}}$ **Directional derivative along with the tangent=grad.** $\hat{e}$ = $\frac{18}{\sqrt{14}}$. **(5Marks)**
	b) In robotics path planning, a robot is moving in a 3-dimensional environment modeled by the potential field $\vec{F}=(y^2\cos x+z^3)\vec{i}+(2y\sin x-4)\vec{j}+3xz^2\vec{k}$. **Show that this field represents a conservative artificial potential field, and compute the scalar potential function that the robot would use for energy-efficient navigation.** **Solution:**

Given $\vec{F} = (y^2 \cos x + z^3)\vec{i} + (2y \sin x - 4)\vec{j} + 3xz^2\vec{k}$

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 \cos x + z^3 & 2y \sin x - 4 & 3xz^2 \end{vmatrix}$$

$$\nabla \times \vec{F} = = \vec{i}[0-0] - \vec{j}\left[3z^2 - 3z^2\right] + \vec{k}[2y\cos x - 2y\cos x] = 0\vec{i} - 0\vec{j} + 0\vec{k} = \vec{0}$$

Hence $\vec{F}$ is irrotational

Finding Scalar Potential

$\vec{F} = \nabla\phi$

$$\Rightarrow (y^2 \cos x + z^3)\vec{i} + (2y \sin x - 4)\vec{j} + 3xz^2\vec{k} = \vec{i}\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y} + \frac{\partial\phi}{\partial z}$$

The scalar potential function = $\phi = y^2 \sin x + xz^3 - 4y + C$ **.** **(5Marks)**

4. **a) A virtual reality (*VR*) haptic device applies a force to a user's hand, modeled by the vector field** $\vec{F} = (x^2 - y^2 + x)\vec{i} - (2xy + y)\vec{j}$**, where *(x,y)* represents the position of the hand in meters in a *2D* plane. If the user moves their hand along the straight line path from point *A(0,0)* to point *B(1,1)*. Find the work done by the force along path** $y^2 = x$ **.**

Solution:

Given $\vec{F} = (x^2 - y^2 + x)\vec{\imath} - (2xy + y)\vec{\jmath}$

$d\vec{r} = dx\vec{\imath} + dy\vec{\jmath}$

$\vec{F} \cdot d\vec{r} = (x^2 - y^2 + x)dx - (2xy + y)dy$

Given $y^2 = x \Rightarrow 2ydy = dx$

Along the curve C, y varies from 0 to 1.

$$\int_c \vec{F} \cdot d\vec{r} = \int_0^1((y^2)^2 - y^2 + y^2)\, 2ydy - (2(y^2)y + y)dy$$

$$= \int_0^1(2y^5 - 2y^3 + 2y^3 - 2y^3 - y)\, dy$$

$$= \int_0^1(2y^5 - 2y^3 - y)\, dy$$

$$= \left[2\frac{y^6}{6} - 2\frac{y^4}{4} - \frac{y^2}{2}\right]_0^1$$

$$= \frac{2}{6} - \frac{2}{4} - \frac{1}{2} = -\frac{2}{3}$$

(5Marks)

b) Using Green's theorem evaluate, $\int_C (y - \sin x)dx + \cos x\, dy$**, where *C* is the plane triangle bounded by the lines** $y = 0,\ x = \frac{\pi}{2},\ y = \left(\frac{2}{\pi}\right)x$ **.**

Solution:

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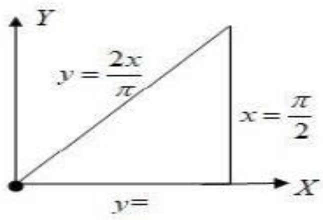
Solution: Green's theorem states that $\int_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$

Given $\int_C (y - \sin x)dx + \cos x\, dy$



$P = y - \sin x \Rightarrow \frac{\partial P}{\partial y} = 1$; $Q = \cos x \quad \Rightarrow \frac{\partial Q}{\partial x} = -\sin x$

$$\int_C (y - \sin x)dx + \cos x\, dy = \iint_R (-\sin x - 1)\, dx\, dy$$

$$= \int_0^1 \int_{\frac{\pi y}{2}}^{\frac{\pi}{2}} (-\sin x - 1)\, dx\, dy$$

$$= \int_0^1 [\cos x - x]_{\frac{\pi y}{2}}^{\frac{\pi}{2}} dy = \int_0^1 \left[\left(\cos\frac{\pi}{2} - \frac{\pi}{2}\right) - \left(\cos\frac{\pi y}{2} - \frac{\pi y}{2}\right)\right] dy$$

$$= \left[-\frac{\pi y}{2} - \frac{\sin\frac{\pi y}{2}}{\frac{\pi}{2}} + \frac{\pi y^2}{4} \right]_0^1 = -\left(\frac{\pi}{4} + \frac{2}{\pi}\right)$$

(5Marks)

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5. **An electric circuit consists of $R = 40\Omega$, $L = 1H$, $C = 16 \times 10^{-4} F$ .The electrical oscillations in the circuit are free. If the initial charge and current are both zero. Determine the charge and current at time.**

Solution: **The differential equation is** $\frac{d^2q}{dt^2} + 40\frac{dq}{dt} + 625q = 0$

The auxiliary equation is $m^2 + 40m + 625 = 0$

The roots are $m = -20 \pm i\,15$

The complementary equation is $q(t) = e^{-20t}\left(A\cos 15t + B\sin 15t\right)$**.**

Initial conditions q(0)=0 and i(0)=q'(0)=0

From q(0)=0; A=0.

Compute q'(t); $q'(t) = e^{-20t}\left((-15A - 20B)\sin 15t + (-20A + 15B)\cos 15t\right)$**;**

At t=0; q'(0)=-20A+15B=0; B=0

Thus A=B=0, so the solution is the trivial one.

$q(t) = 0, \quad \text{for all } t, \quad i(t) = q'(t) = 0 \quad \text{for all } t$

With zero initial charge, zero initial current and no applied *emf* the circuit contains no stored energy, so it remains at rest for all time. **(10Marks)**