VIT
Vellore Institute of Technology

REG.NO.:

SCHOOL OF ADVANCED SCIENCES
CONTINUOUS ASSESSMENT TEST - II
FALL SEMESTER 2025-2026

SLOT: G1+TG1

Programme Name & Branch : B.Tech., Common to all branches
Course Code and Course Name : BAMAT101- Multivariable Calculus and Differential Equations
Faculty Name(s) : Common to G1
Class Number(s) : Common to G1
Date of Examination : 11.10.2025
Exam Duration : 90 minutes **Maximum Marks: 50**

Answer All Questions

Q. No	Question	M	CO	BL
1.	A composite block of insulating material occupies the region in 3D space defined by $1 \le x \le 3, -x \le y \le 2x, and\ 0 \le z \le x + y^2$. At any point (x, y, z), the volumetric heat capacity is proportional to the x-coordinate i.e. $f(x, y, z) = x$. Evaluate the triple integral $\int\int\int_V f(x, y, z)dV$ representing the total heat capacity of the block.	10	2	3
2.	In medical scanning techniques, such as MRI or CT, volumetric reconstruction of organs is often performed using mathematical functions that represent density or composition inside the body. Suppose the cross-sectional density distribution inside a perfectly spherical organ (like an eyeball) of radius 1 unit is given by the function $f(x, y) = \frac{1}{\sqrt{1-x^2-y^2-z^2}}$ at any point (x, y, z) inside the organ. Calculate the total density measure inside the organ i.e. $\iiint_C \frac{1}{\sqrt{1-x^2-y^2-z^2}} dxdydz$ where C is interior of the spherical organ $x^2 + y^2 + z^2 = 1$ by transforming into spherical coordinates.	10	2	3
3.	In fluid mechanics, the circulation of a velocity field around a closed curve is often computed using Green's theorem instead of evaluating the line integral directly. Suppose the velocity field of a fluid in the plane is given by $\vec{F} = x^2(1 + y)\vec{i} + (x^3 + y^3)\vec{j}$. Let the boundary C be the square plate in the plane with corners at $x = \pm 1$ and $y = \pm 1$. Verify Greens theorem for this velocity field.	10	3	3
4.	In the study of fluid dynamics for biomedical engineering, consider a blood flow velocity field within a vessel given by $\vec{F} = 3yz\vec{i} + 2zx\vec{j} + 4xy\vec{k}$ where x, y, z correspond to coordinates inside a vessel. Prove whether the velocity field describes a flow without local spinning (i.e., test if it is irrotational) and discuss the physical implications for blood flow. Now consider the modified field $(x^2yz^3)\vec{F}$ which could represent the velocity distribution for blood under the influence of a localized medication effect. Determine whether this new field is irrotational and solenoidal, and interpret the results in the context of mass conservation in biomedical flows. For the irrotational field, find the scalar potential function.	10	3	3
5.	The charge $q(t)$ on the capacitor at any time t in the LCR circuit, with the resistance $6\ \Omega$, inductance $0.5\ H$, capacitance $0.02\ F$, with zero emf. Find the charge $q(t)$ and the current $i(t)$ at time t when: (i) the initial current is one and charge on the capacitor is zero and (ii) the initial current is zero and charge on the capacitor is one.	10	4	3

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