REG.NO.:

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

School of Computer Science and Engineering

SLOT: F1+TF1

CONTINUOUS ASSESSMENT TEST - I
WINTER SEMESTER 2024-2025

Programme Name & Branch:Btech, CSE
Course Code and Course Name : BCSE304L and Theory of Computation
Faculty Name(s) : KANAGARAJ R, MADIAJAGAN M, SATHIYA KUMAR C , KARTHIK G M, BASKARAN P, PARTHASARATHY G , SARITHA MURALI , RAJARAJAN G , SHALINI L, UMA PRIYA D , BOOMINATHAN P, LAKSHMANAN K, BHAWANA TYAGI , BHUVANESWARI M, IYAPPAN P, ISLABUDEEN M, PRAKASH M, SATHYA K, ADRIJA BHATTACHARYA, DEBI PRASANNA ACHARJYA, K.Krishna Rani Samal,SUGANTHINI C

Class Number(s) :VL2024250501633,VL2024250501615, VL2024250501619, VL2024250501625, VL2024250501637,VL2024250501639,VL2024250501635,VL2024250501653,VL2024250501631,VL2024250501643,VL2024250501617,VL2024250501651,VL2024250501649,VL2024250501645,VL2024250501627,VL2024250501647,VL2024250501629,VL2024250501641,VL2024250501623, VL2024250501613, VL2024250501621, VL2024250501655
Date of Examination : 01-Feb-2025, 09:30 AM - 11:00 AM
Exam Duration : 90 minutes Maximum Marks: 50

General instruction(s):

- Answer All Questions
- M - Max mark; CO – Course Outcome; BL – Blooms Taxonomy Level (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create)

Q. No	Question	M	CO	BL
1	a)Consider the Fibonacci sequence $\{x_n\}_{n=1}^{\infty}$, defined by the relations $x_1 = 1, x_2 = 1$ and $x_n = x_{n-1} + x_{n-2}$ for $n \geq 3$. I. Use an extended Principle of Mathematical Induction in order to show that for n ≥ 1, $x_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$ II. Compute x_{20} Solution:	5 5	4	2

For any $n \geq 1$, let P_n be the statement that

$$x_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right].$$

Base Case. The statement P_1 says that

$$\begin{aligned} x_1 &= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^1 - \left(\frac{1-\sqrt{5}}{2}\right)^1\right] \\ &= \frac{1}{\sqrt{5}}\left[\frac{1+\sqrt{5}}{2} - \frac{1-\sqrt{5}}{2}\right] \\ &= \frac{1}{\sqrt{5}}\left[\frac{1+\sqrt{5}-1+\sqrt{5}}{2}\right] \\ &= \frac{1}{\sqrt{5}}\left[\frac{2\sqrt{5}}{2}\right] \\ &= 1, \end{aligned}$$

which is true. The statement P_2 says that

$$x_2 = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^2 - \left(\frac{1-\sqrt{5}}{2}\right)^2\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+2\sqrt{5}+5}{4}\right) - \left(\frac{1-2\sqrt{5}+5}{4}\right)\right]$$
$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+2\sqrt{5}+5-1+2\sqrt{5}-5}{4}\right)\right]$$
$$= \frac{1}{\sqrt{5}}\left[\frac{4\sqrt{5}}{4}\right]$$
$$= 1,$$

which is again true.

Inductive Step. Fix $k \geq 1$, and suppose that P_k and P_{k+1} both hold, that is,

$$x_k = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^k - \left(\frac{1-\sqrt{5}}{2}\right)^k\right],$$

and

$$x_{k+1} = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{k+1}\right].$$

It remains to show that P_{k+2} holds, that is, that

$$x_{k+2} = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k+2} - \left(\frac{1-\sqrt{5}}{2}\right)^{k+2}\right].$$

$$x_{k+2} = x_k + x_{k+1}$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^k - \left(\frac{1-\sqrt{5}}{2}\right)^k\right] + \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{k+1}\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^k + \left(\frac{1+\sqrt{5}}{2}\right)^{k+1} - \left(\frac{1-\sqrt{5}}{2}\right)^k - \left(\frac{1-\sqrt{5}}{2}\right)^{k+1}\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^k \left(1+\frac{1+\sqrt{5}}{2}\right) - \left(\frac{1-\sqrt{5}}{2}\right)^k \left(1+\frac{1-\sqrt{5}}{2}\right)\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^k \left(\frac{3+\sqrt{5}}{2}\right) - \left(\frac{1-\sqrt{5}}{2}\right)^k \left(\frac{3-\sqrt{5}}{2}\right)\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^k \left(\frac{6+2\sqrt{5}}{4}\right) - \left(\frac{1-\sqrt{5}}{2}\right)^k \left(\frac{6-2\sqrt{5}}{4}\right)\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^k \left(\frac{1+2\sqrt{5}+5}{4}\right) - \left(\frac{1-\sqrt{5}}{2}\right)^k \left(\frac{1-2\sqrt{5}+5}{4}\right)\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^k \left(\frac{1+\sqrt{5}}{2}\right)^2 - \left(\frac{1-\sqrt{5}}{2}\right)^k \left(\frac{1-\sqrt{5}}{2}\right)^2\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k+2} - \left(\frac{1-\sqrt{5}}{2}\right)^{k+2}\right].$$

Therefore P_{k+2} holds.

Thus by the principle of mathematical induction, for all $n \geq 1$, P_n holds.

ii)

$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765$

b) Formally define DFA machine M and language L(M) accepted by M. [2M]

ANS:

A **Deterministic Finite Automaton (DFA)** is a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$, where:

1. Q: A finite set of states.
2. Σ: A finite set of input symbols (alphabet).
3. δ: A transition function, $\delta : Q \times \Sigma \to Q$, mapping state-symbol pairs to a unique next state.
4. $q_0 \in Q$: The start state.
5. $F \subseteq Q$: A set of accepting (final) states.

The **language** $L(M)$ accepted by M is defined as:

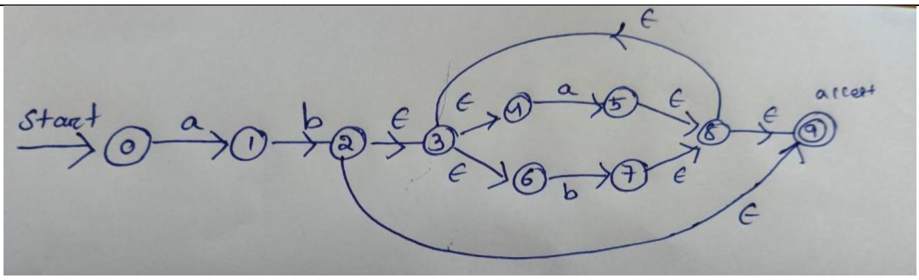
$$L(M) = \{w \in \Sigma^* \mid \delta^*(q_0, w) \in F\},$$

where δ^* is the extended transition function, defined recursively for strings in Σ^*.

c) Correlate formal grammars, languages with their respective computational models. [3M]

Formal grammars, languages, and computational models are tightly interconnected in the **Chomsky Hierarchy**:

1. **Type 0 (Unrestricted Grammar)**:
   - Language: Recursively Enumerable Languages.
   - Computational Model: **Turing Machines**.
   - Grammar: Rules of the form $\alpha \to \beta$, where $\alpha, \beta \in (V \cup \Sigma)^*$ and $\alpha \neq \epsilon$.
2. **Type 1 (Context-Sensitive Grammar)**:
   - Language: Context-Sensitive Languages.
   - Computational Model: **Linear Bounded Automata (LBA)**.
   - Grammar: Rules of the form $\alpha A \beta \to \alpha \gamma \beta$, where $\gamma \neq \epsilon$.
3. **Type 2 (Context-Free Grammar)**:
   - Language: Context-Free Languages.
   - Computational Model: **Pushdown Automata (PDA)**.
   - Grammar: Rules of the form $A \to \gamma$, where $A \in V$ and $\gamma \in (V \cup \Sigma)^*$.
4. **Type 3 (Regular Grammar)**:
   - Language: Regular Languages.
   - Computational Model: **Finite Automata (DFA or NFA)**.
   - Grammar: Rules of the form $A \to aB$ or $A \to a$, where $A, B \in V$ and $a \in \Sigma$.

	This hierarchy demonstrates how grammars define languages and how computational models process these languages. For instance: • Regular languages can be recognized by finite automata and are generated by regular grammars. • Context-free languages are processed by pushdown automata and are generated by context-free grammars.			
2.	 I. Convert the given ε-NFA to its equivalent DFA. Draw the DFA diagram. II. Determine whether minimization of the resulting DFA is possible. Explain your answer.	10	1	3

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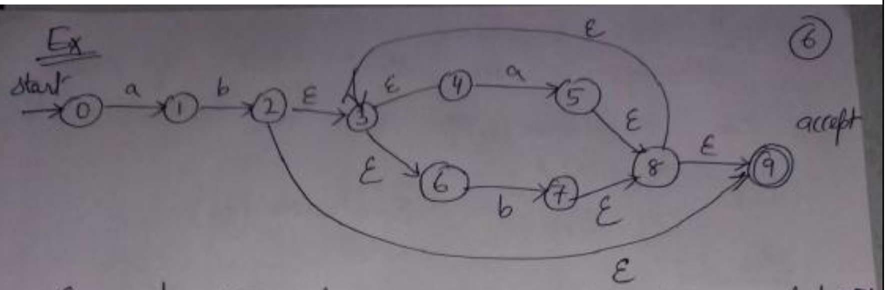
ANS:

Ex



Convert the above ε-NFA to its equivalent DFA

Sol:→

Step 1 :→ Identify the start state of DFA.

Since 0 is the start state of ε-NFA,

∴ ε-closure (0) is the start state of

i.e, ε-closure (0) = {0} — (A)

Consider state A

Give inputs

$\delta(A, a) = \varepsilon\text{-closure}(\delta_E(A, a))$

$= \varepsilon\text{-closure}(\delta_E(0, a))$

$= \{1\}$ — B

$\delta(A, b) = \varepsilon\text{-closure}\ \delta_E(0, b)$

$= \varepsilon\text{-closure}(\delta_E(0, b))$

$= \{\phi\}$

Consider state B

$\delta(B, a) = \varepsilon\text{-closure}(\delta_E(B, a))$

$= \varepsilon\text{-closure}(\delta_E(1, a)) = \{$

$\delta(B, b) = \varepsilon\text{-closure}(\delta_E(B, b))$ (7)

$= \varepsilon\text{-closure}(\delta_E(1, b))$

$= \varepsilon\text{-closure}(\{2\})$

$= \{2, 3, 4, 6, 9\}$ — (C)

<u>Consider state C</u>

$\delta(C, a) = \varepsilon\text{-closure}(\delta_E(C, a))$

$= \varepsilon\text{-closure}(\delta_E(\{2, 3, 4, 6, 9\}, a)$

$= \varepsilon\text{-closure}\{5\}$

$= \{5, 8, 9, 3, 4, 6\}$

$= \{3, 4, 5, 6, 8, 9\}$ — (D)

$\delta(C, b) = \varepsilon\text{-closure}(\delta_E(C, b))$

$= \varepsilon\text{-closure}(\delta_E(\{2, 3, 4, 6, 9\}, b)$

$= \varepsilon\text{-closure}(\{7\})$

$= \{7, 8, 9, 3, 4, 6\}$ (E)

$= \{3, 4, 6, 7, 8, 9\}$ — (E)

<u>Consider state D</u>

$\delta(D, a) = \varepsilon\text{-closure}(\delta_E\{3, 4, 5, 6, 8, 9\}, a)$

$= \varepsilon\text{-closure}\{5\}$

$= \{5, 8, 9, 3, 4, 6\}$

$\delta(D,b) = \varepsilon\text{-closure}(\delta_E(D,b))$ (8)

$= \varepsilon\text{-closure}(\delta_E(\{3,4,5,6,8,9\},b))$

$= \varepsilon\text{-closure}\{7\}$

$= \{7,8,9,3,4,6\}$

$= \{3,4,6,7,8,9\}$ — E

Consider state E

$\delta(E,a) = \varepsilon\text{-closure}(\delta_E(E,a))$

$= \varepsilon\text{-closure}(\delta_E\{3,4,6,7,8,9\},a)$

$= \varepsilon\text{-closure}\{5\}$

$= \{5,8,9,3,4,6\}$

$= \{3,4,5,6,8,9\}$ —(D)

$\delta(E,b) = \varepsilon\text{-closure}(\delta_E(E,b))$

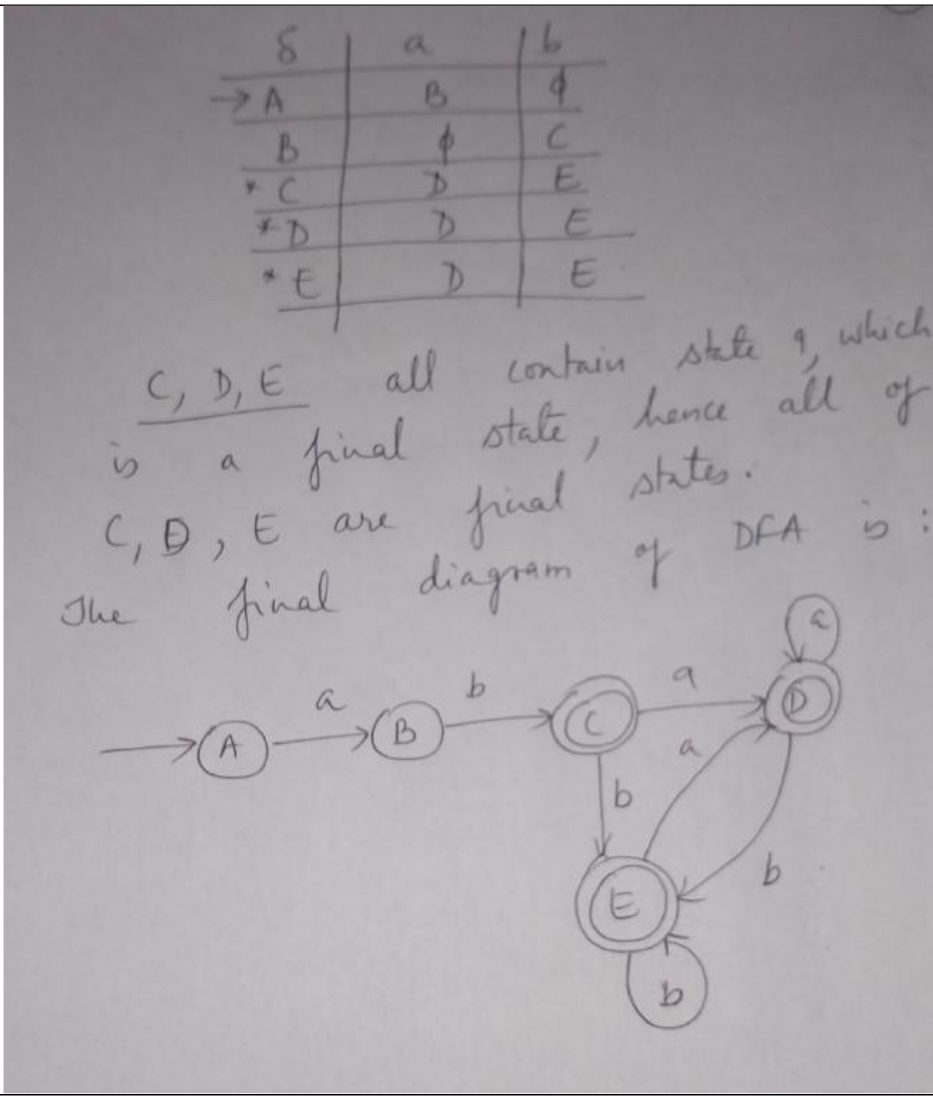
$= \varepsilon\text{-closure}(\delta_E\{3,4,6,7,8,9\},b)$

$= \varepsilon\text{-closure}\{7\}$

$= \{7,8,9,3,4,6\}$

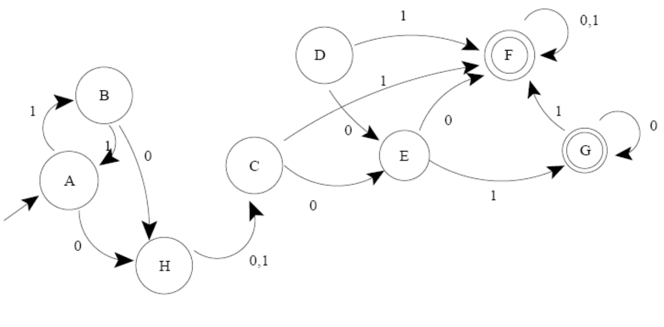
$= \{3,4,6,7,8,9\}$ —(E)

Since there are no new states, we can draw the TT.

<table>
<tr><td></td><td></td><td>


δ	a	b
→A	B	ϕ
B	ϕ	C
*C	D	E
*D	D	E
*E	D	E

C, D, E all contain state q, which is a final state, hence all of C, D, E are final states.
The final diagram of DFA is:

</td><td></td><td></td><td></td></tr>
<tr><td>3</td><td>I.
II.</td><td>Consider the following DFA. Determine whether it is already a minimized DFA by demonstrating the equivalence steps. [8M]
If it can be further minimized, draw the diagram of the final minimized DFA. [2M]


Solution: Check for pairs with one state final and one not</td><td>10</td><td>1</td><td>3</td></tr>
</table>

b							
c							
d							
e							
f	ϵ	ϵ	ϵ	ϵ	ϵ		
g	ϵ	ϵ	ϵ	ϵ	ϵ		
h						ϵ	ϵ
	a	b	c	d	e	f	g

First iteration of main loop:

b							
c	1	1					
d	1	1					
e	0	0	0	0			
f	ϵ	ϵ	ϵ	ϵ	ϵ		
g	ϵ	ϵ	ϵ	ϵ	ϵ		
h			1	1	0	ϵ	ϵ
	a	b	c	d	e	f	g

Second iteration of main loop:

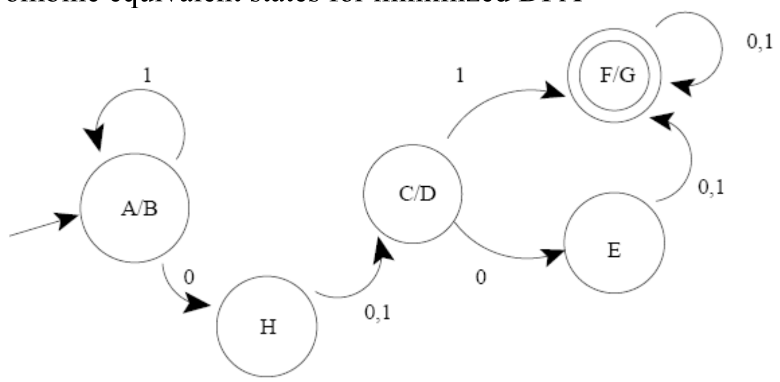
b							
c	1	1					
d	1	1					
e	0	0	0	0			
f	ϵ	ϵ	ϵ	ϵ	ϵ		
g	ϵ	ϵ	ϵ	ϵ	ϵ		
h	1	1	1	1	0	ϵ	ϵ
	a	b	c	d	e	f	g

Third iteration makes no changes

Blank cells are equivalent pairs of states

b							
c	1	1					
d	1	1					
e	0	0	0	0			
f	ϵ	ϵ	ϵ	ϵ	ϵ		
g	ϵ	ϵ	ϵ	ϵ	ϵ		
h	1	1	1	1	0	ϵ	ϵ
	a	b	c	d	e	f	g

Combine equivalent states for minimized DFA



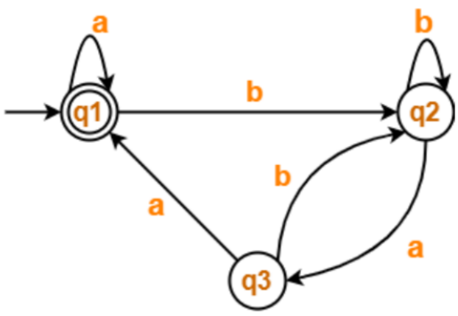
4 | I. Write the regular expression for the language accepting all the string of odd length over ∑ = {a, b}. [3M] | 10 | 2 | 3

ANS:

a((a+b)(a+b))* + b((a+b)(a+b))*

This regular expression describes a string that starts with either an "a" or a "b", followed by any number of pairs of "a" and "b" **(represented** as (a+b)), and then ends with a star (denoting zero or more occurrences) of (a+b)(a+b), which ensures that the length of the string is odd.

II. b)Find regular expression for the following DFA. [7M]



Solution-

Step-01:

Form a equation for each state-

q1 = ∈ + q1.a + q3.a ……(1)

q2 = q1.b + q2.b + q3.b ……(2)

q3 = q2.a …….(3)

Step-02:

Bring final state in the form R = Q + RP.

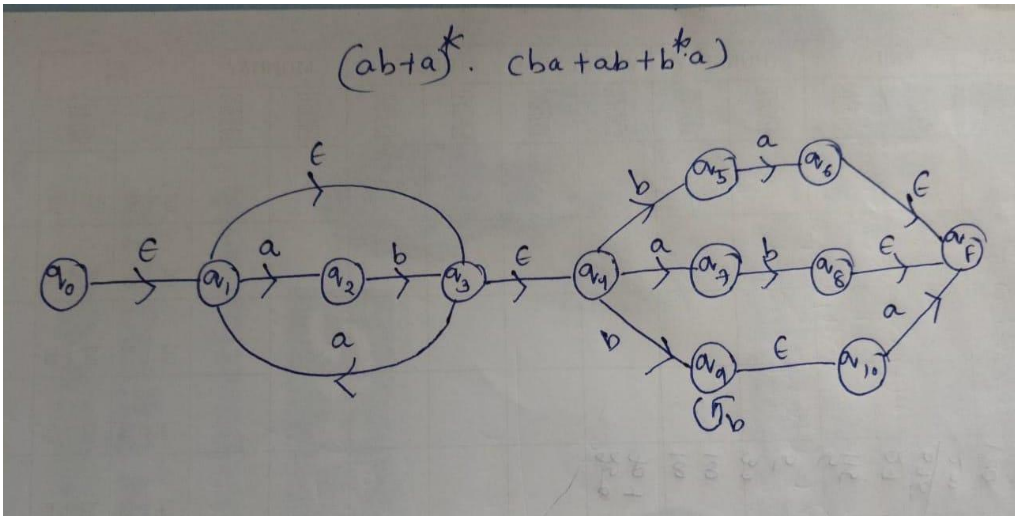
Using (3) in (2), we get-

q2 = q1.b + q2.b + q2.a.b

q2 = q1.b + q2.(b + a.b) …….(4)

Using Arden's Theorem in (4), we get-

q2 = q1.b.(b + a.b)* …….(5)

	Using (5) in (3), we get- q3 = q1.b.(b + a.b)*.a …….(6) Using (6) in (1), we get- q1 = ∈ + q1.a + q1.b.(b + a.b)*.a.a q1 = ∈ + q1.(a + b.(b + a.b)*.a.a) …….(7) Using Arden's Theorem in (7), we get- q1 = ∈.(a + b.(b + a.b)*.a.a)* q1 = (a + b.(b + a.b)*.a.a)* Thus, Regular Expression for the given DFA = (a + b(b + ab)*aa)*			
5	I. Obtain an Epsilon NFA for regular expression (ab+a)* (ba+ab+b*a) over Σ={a,b} [5M]  II. Prove **ε + 1*(011)*(1*(011)*)* =(1 + 011)*** [5M] **Solution: R = ε + PP* ,** where **P = 1*(011)*** **= P*** using I9 **= (Q*S*)*** where **Q=1**, **S=011**	10	2	2

	= **(Q + S)*** using $\mathbf{I}_{11}$ = **(1 + 011)***				