REG.NO.:

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SCHOOL OF ADVANCED SCIENCES
CONTINUOUS ASSESSMENT TEST - I
WINTER SEMESTER 2024-2025

SLOT: D2+TD2

Programme Name & Branch :B.Tech
Course Code and Course Name : BMAT202L (Probability and Statistics)
Faculty Name(s) : Common question paper for D2 slot
Class Number(s) : Common question paper for D2 slot
Date of Examination : 30-01-2025
Exam Duration : 90 minutes **Maximum Marks: 50**

General instruction(s):

- Answer All Questions
- M - Max mark; CO – Course Outcome; BL – Blooms Taxonomy Level (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create)

Q. No	Question	M	CO	BL
1.	In the frequency distribution of 170 families given below, the number of families corresponding to expenditure group 20-30 and 40-50 is missing in the table. However the median is known to be 35. Find the missing frequencies. Expenditure: 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 No.of families: 10 | 20 | ? | 40 | ? | 25 | 15	10	1	1
2.	The score of two players A and B in ten innings during a certain season are: A: 32 | 28 | 47 | 63 | 71 | 39 | 10 | 60 | 96 | 14 B: 19 | 31 | 48 | 53 | 67 | 90 | 10 | 62 | 40 | 80 Find which of the two players A, B is more consistent in scoring.	10	1	2
3.	A random variable X has the following probability distribution. X: 0 | 1 | 2 | 3 | 4 | 5 | 6 P(X=x): a | 3a | 5a | 7a | 9a | 11a | 13a (i) Find the value of 'a' (ii) Find the c.d.f F(x) of X (iii) Evaluate $p(X \geq 4)$ (iv) Evaluate $p(X < 5)$ (v) Evaluate $p(3 \leq X \leqq 6)$ (vi) Find $p(X = 5)$ using F(x).	10	2	1
4.	The joint density for the random variable (X,Y),where X is the temperature change and Y is the proportion of the spectrum that shifts for a certain atomic particle, is $f(x,y) = \begin{cases} 10xy^2, & 0 < x < y < 1 \\ 0, otherwise \end{cases}$ (i) Find $f_x(x), f_y(y)$ and $f(y/x)$ (ii) Find the probability that the spectrum shifts more than half of the total observations, given that the temperature is increased by 0.25 unit.	10	2	2
5.	Compute the coefficient of correlation between X and Y from the following data and comment on it. X: 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 Y: 20 | 30 | 37 | 50 | 56 | 78 | 89 | 100 | 120 | 110	10	3	2

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Slot - D2+TD2 CAT 1 - KEY

Sub: Course: BMAT202L - Probability & statistics

1. Let the missing frequencies of the expenditure groups 20-30 and 40-50 be f_1 and f_2 respectively. Given Median = 35 and total frequency = 170.

Class	f	cumulative frequency
0-10	10	10
10-20	20	30
20-30	f_1	$30+f_1 = m$
30-40 median class	40 = f	$70+f_1$
40-50	f_2	$70+f_1+f_2$
50-60	25	$95+f_1+f_2$
60-70	15	$110+f_1+f_2$
	170	

From the table, we have

$L = 30$, $f = 40$, $m = 30+f_1$

$\frac{N}{2} = \frac{170}{2} = 85$ & $C = 10$

$$\text{Median} = L + \left[\frac{\frac{N}{2} - m}{f}\right] C$$

$$35 = 30 + \left[\frac{85 - 30 - f_1}{40}\right] 10$$

$\boxed{f_1 = 35}$, Total frequency $N = 170$

$\therefore 170 = 110 + f_1 + f_2$

$\therefore \boxed{f_2 = 25}$

(2) Calculation of co-efficient of variation

X	$x-\bar{x}$	$(x-\bar{x})^2$	Y	$y-\bar{y}$	$(y-\bar{y})^2$
32	-14	196	19	-31	961
28	-18	324	31	-19	361
47	1	1	48	-2	4
63	17	289	53	3	9
71	25	625	67	17	289
39	-7	49	90	40	1600
10	-36	1296	10	-40	1600
60	14	196	62	12	144
96	50	2500	40	-10	100
14	-32	1024	80	30	900
$\Sigma X = 460$	0	6500	$\Sigma Y = 500$	0	5968

$$\bar{x} = \frac{\Sigma x}{10} = \frac{460}{10} = 46$$

$$\bar{y} = \frac{\Sigma y}{10} = \frac{500}{10} = 50$$

$$\sigma_A^2 = \frac{\Sigma (x_i - \bar{x})^2}{N} = \frac{6500}{10} = 650$$

$$\sigma_B^2 = \frac{\Sigma (y_i - \bar{y})^2}{N} = \frac{5968}{10} = 596.8$$

$$\sigma_A = \sqrt{650} = 25.5$$

$$\sigma_B = \sqrt{596.8} = 24.43$$

$$C.V_{(A)} = \frac{\sigma_A}{\bar{x}} \times 100 = 55.43$$

$$C.V_{(B)} = \frac{\sigma_B}{\bar{y}} \times 100 = 48.86$$

∴ For player A:

$\bar{x} = 46$

$\sigma_A = 25.5$

$C.V_{(A)} = 55.43$

Player B:

$\bar{y} = 50$

$\sigma_B = 24.43$

$C.V_{(B)} = 48.86.$

∴ player A having more consistent than player B

(3)

X:	0	1	2	3	4	5	6
P(X=x):	a	3a	5a	7a	9a	11a	13a

(i) $\sum P(x) = 1$

$\Rightarrow P(x=0) + P(x=1) + \cdots + P(x=6) = 1$

$a + 3a + 5a + 7a + 9a + 11a + 13a = 1$

$49a = 1 \Rightarrow \boxed{a = \frac{1}{49}}$

(ii) The c.d.f F(x)

X:	0	1	2	3	4	5	6
P(x):	$\frac{1}{49}$	$\frac{3}{49}$	$\frac{5}{49}$	$\frac{7}{49}$	$\frac{9}{49}$	$\frac{11}{49}$	$\frac{13}{49}$
F(x):	$\frac{1}{49}$	$\frac{4}{49}$	$\frac{9}{49}$	$\frac{16}{49}$	$\frac{25}{49}$	$\frac{36}{49}$	$\frac{49}{49} = 1$

Since $F(x) = P(X < x)$

(iii) $P(X \geq 4) = P(X=4) + P(X=5) + P(X=6)$

$= 9a + 11a + 13a = 33a = \frac{33}{49}$

(iv) $P(X<5) = 1 - P(X \geq 5)$

$= 1 - [P(X=5) + P(X=6)]$

$= 1 - [11a + 13a]$

$= 1 - [24a]$

$= 1 - \frac{24}{49}$

$P(X<5) = \frac{25}{49}$

(v) $P(3 \le x \le 6) = P(x=3) + P(x=4) + P(x=5) + P(x=6)$

$= \frac{40}{49}$

(vi) $P(x=5)$ using $F(x)$

$\therefore P(x=5) = F(5) - F(4)$

$= \frac{36}{49} - \frac{25}{49}$

$= \frac{11}{49}$

$=$

(4) Given joint density for (x,y) is

$f(x,y) = 10xy^2 \quad 0 < x < y < 1$

$= 0$, elsewhere.

(i) The marginal density function of x is:

(a) $f_x(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_{x}^{1} 10xy^2\,dy =$

$\therefore f_x(x) = \frac{10x(1-x^3)}{3}, \quad 0 < x < 1$

(b) $f_y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_{0}^{y} 10xy^2\,dx = 5y^4, \quad 0 < y < 1$

(ii) $f(y|x) = \frac{f(x,y)}{f_x(x)}$

$= \frac{3y^2}{1-x^3}, \quad 0 < x < y < 1$

(5)

(ii) $P(Y > \frac{1}{2} / x = 0.25) = \int_{y_2}^{1} f(y/x=0.25)\, dy$

$$= \int_{y_2}^{1} \frac{3y^2}{1-(0.25)^3}\, dy$$

$$= 0.89$$

(5) *: $\Sigma x = 190$ $\Sigma y = 690$

$\bar{x} = \frac{\Sigma x}{10} = 19$ $\bar{y} = \frac{\Sigma y}{10} = 69$

x	y	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
10	20	-9	-49	81	2401	441
12	30	-7	-39	49	1521	273
14	37	-5	-32	25	1024	160
16	50	-3	-19	9	364	57
18	56	-1	-13	1	169	13
20	78	1	9	1	81	9
22	89	3	20	9	400	60
24	100	5	31	25	961	155
26	120	7	51	49	2601	357
28	110	9	41	81	1681	369
190	690	0	0	330	11200	1894

Correlation coeffi - $f = \frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma (x_i - \bar{x})^2 \cdot \Sigma (y_i - \bar{y})^2}} = 0.985$

The correlation coeff. b/n annual expenditure & annual sales revenue is 0.985