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SCHOOL OF MECHANICAL ENGINEERING
CONTINUOUS ASSESSMENT TEST (CAT) -II
WINTER SEMESTER 2024-2025

Programme Name & Branch	**: B. Tech Mechanical Engineering**
Course Code	**: BMEE306L**
Course Name	**: Computer Aided Design & Finite Element Analysis**
Faculty Name(s)	**: Dr. P. Kuppan, Dr. M. Murugan, Dr. R. Manoharan, Dr. A.Vinoth Jebaraj, Dr. A. Deepa,**
Class Number(s)	**: VL2024250503569, 3570, 3571, 3572, 3573**
Date of Examination	**: 22.03.2025**
Exam Duration	**: 90 minutes**

Maximum Marks: 50

ANSWER KEY

Q. No	Question
1.	Find a two term Galerkin solution to the differential equation $\frac{d^2y}{dx^2}$ -x³-5=0 for $0 \le x \le 1$ With boundary conditions y(0) = 0 and y(1) =0

① $\frac{d^2y}{dx^2} - x^3 - 5 = 0 \qquad 0 \le x \le 1$

Bc's $y(0) = 0 \qquad y(1) = 0$

Assume a cubic polynomial trial solution to get two term Galerkin Solution.

$y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$y(0) = 0 \Rightarrow a_0 = 0$

$y(1) = 0 \Rightarrow a_1 = -a_2 - a_3$

$y(x) = (-a_2 - a_3)x + a_2 x^2 + a_3 x^3$

$\boxed{y(x) = a_2(x^2 - x) + a_3(x^3 - x)}$

Substitute the above trial soln in GDE

Residue $\boxed{'R_d' = 2a_2 + 6a_3 x - x^3 - 5}$

Galerkin's WR statement

$\int_0^1 W(x) R_d(x)\, dx = 0$

$\int_0^1 (x^2 - x)(2a_2 + 6a_3 x - x^3 - 5)\, dx = 0 \quad -①$

$\int_0^1 (x^3 - x)(2a_2 + 6a_3 x - x^3 - 5)\, dx = 0 \quad -②$

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From eqn 1

$$\int_0^1 \left(2a_2x^2 + 6a_3x^3 - x^5 - 5x^2 - 2a_2x - 6a_3x^2 + x^4 + 5x\right) dx = 0$$

$$\left[\frac{2a_2x^3}{3} + \frac{6a_3x^4}{4} - \frac{x^6}{6} - \frac{5x^3}{3} - \frac{2a_2x^2}{2} - \frac{6a_3x^3}{3} + \frac{x^5}{5} + \frac{5x^2}{2}\right]_0^1 = 0$$

$$\frac{2}{3}a_2 + \frac{3}{2}a_3 - \frac{1}{6} - \frac{5}{3} - a_2 - 2a_3 + \frac{1}{5} + \frac{5}{2} = 0$$

$$\boxed{-0.33a_2 - 0.5a_3 + 0.8666 = 0} \quad \text{— (1)}$$

From eqn 2

$$\int_0^1 \left(2a_2x^3 + 6a_3x^4 - x^6 - 5x^3 - 2a_2x - 6a_3x^2 + x^4 + 5x\right) dx = 0$$

$$\left[\frac{2a_2x^4}{4} + \frac{6a_3x^5}{5} - \frac{x^7}{7} - \frac{5x^4}{4} - \frac{2a_2x^2}{2} - \frac{6a_3x^3}{3} + \frac{x^5}{5} + \frac{5x^2}{2}\right]_0^1 = 0$$

$$\frac{a_2}{2} + \frac{6}{5}a_3 - \frac{1}{7} - \frac{5}{4} - a_2 - 2a_3 + \frac{1}{5} + \frac{5}{2} = 0$$

$$-0.5a_2 - 0.8a_3 + 1.307 = 0 \quad \text{— (2)}$$

Solving 1 and 2

$$\boxed{a_2 = 2.84} \quad \boxed{a_3 = -0.142}$$

$$\therefore \boxed{y(x) = 2.84(x^2 - x) - 0.142(x^3 - x)}$$

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2. The spring system shown in figure 1 is loaded with a loads 10N and 20 N at points 1 and 2. If both loads are acting towards right, calculate the deformations of points 1 and 2 using the principle of stationary total potential.

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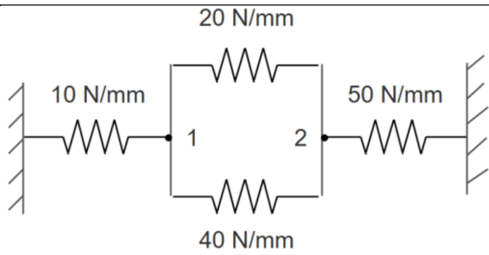

Figure 1

(2) $k_1 = 10\ N/mm$ Strain energy $U = \frac{1}{2}k_1(u_2-u_1)^2 + \frac{1}{2}k_2(u_3-u_2)^2 + \frac{1}{2}k_3(u_4-u_3)^2$

$k_2 = 60\ N/mm$

$k_3 = 50\ N/mm$ Potential of applied forces $V = -F_1u_1 - F_2u_2 - F_3u_3 - F_4u_4$

Total Potential of the System

Acc. to PSTP, $\pi = U + V$

$\frac{\partial \pi}{\partial u_1} = 0$

$k_1(u_2-u_1)(-1) - F_1 = 0$

$k_1u_1 - k_1u_2 - F_1 = 0$

$\frac{\partial \pi}{\partial u_3} = 0$

$k_2(u_3-u_2) + k_3(u_4-u_3)(-1) - F_3 = 0$

$-k_2u_2 + (k_2+k_3)u_3 - k_3u_4 - F_3 = 0$

$\frac{\partial \pi}{\partial u_2} = 0$

$k_1(u_2-u_1) + k_2(u_3-u_2)(-1) - F_2 = 0$

$-k_1u_1 + (k_1+k_2)u_2 - k_2u_3 - F_2 = 0$

$\frac{\partial \pi}{\partial u_4} = 0$

$k_3(u_4-u_3) - F_4 = 0$

$-k_3u_3 + k_3u_4 - F_4 = 0$

$$\begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & k_1+k_2 & -k_2 & 0 \\ 0 & -k_2 & k_2+k_3 & -k_3 \\ 0 & 0 & -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} u_1 \to 0 \\ u_2 \\ u_3 \\ u_4 \to 0 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \to 10\ N \\ F_3 \to 20\ N \\ F_4 \end{Bmatrix}$$

$70u_2 - 60u_3 = 10$ —(1)

$-60u_2 + 110u_3 = 20$ —(2)

Solving (1) and (2) $\boxed{u_2 = 0.56\ mm}$ $\boxed{u_3 = 0.487\ mm}$

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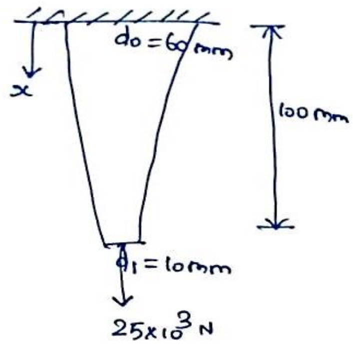
3. A tapered bar is of circular cross section and 100 mm length. The diameter at one end is 60 mm and it is fixed. The other end has a diameter of 10 mm and it carries an axial tensile load of 25 kN. The Young's modulus of the material is 200 GPa.
Discretise the bar in to two elements and find the free end deflection.

$$d(x) = d_0 - (d_0 - d_1)\frac{x}{l}$$

$$= 60 - \left(\frac{50x}{100}\right)$$

$$\boxed{d(x) = 60 - 0.5x}$$



At $x = 25$ mm

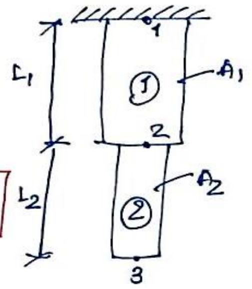
$d_1 = 47.5$ mm

$$\boxed{A_1 = 1772.05 \text{ mm}^2}$$

At $x = 75$ mm

$d_2 = 22.5$ mm

$$\boxed{A_2 = 397.61 \text{ mm}^2}$$



Element 1

$$\frac{AE}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\end{Bmatrix} = \begin{Bmatrix}F_1\\ F_2\end{Bmatrix} \Rightarrow \frac{1772.05 \times 200 \times 10^3}{50}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\end{Bmatrix} = \begin{Bmatrix}F_1\\ F_2\end{Bmatrix}$$

$$10^5\begin{bmatrix}70.88 & -70.88\\ -70.88 & 70.88\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\end{Bmatrix} = \begin{Bmatrix}F_1\\ F_2\end{Bmatrix}$$

Element 2

$$\frac{397.61 \times 200 \times 10^3}{50}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_2\\ u_3\end{Bmatrix} = \begin{Bmatrix}F_2\\ F_3\end{Bmatrix}$$

$$10^5\begin{bmatrix}15.9 & -15.9\\ -15.9 & 15.9\end{bmatrix}\begin{Bmatrix}u_2\\ u_3\end{Bmatrix} = \begin{Bmatrix}F_2\\ F_3\end{Bmatrix}$$

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Global Stiffness matrix

$$10^5\begin{bmatrix} \cancel{70.88} & \cancel{-70.88} & \cancel{0} \\ -70.88 & 86.78 & -15.9 \\ 0 & -15.9 & 15.9\end{bmatrix}\begin{Bmatrix} \cancel{u_1}^{\,0} \\ u_2 \\ u_3\end{Bmatrix} = \begin{Bmatrix}\cancel{F_1}^{\,R_1} \\ \cancel{F_2}^{\,0} \\ 25000\end{Bmatrix}$$

$$10^5\left[86.78\,u_2 - 15.9\,u_3\right] = 0 \quad (1)$$

$$10^5\left[-15.9\,u_2 + 15.9\,u_3\right] = 25000 \quad (2)$$

From (1) $\boxed{u_3 = 5.457\,u_2}$

$$-15.9\,u_2 + 15.9\,(5.457\,u_2) = 0.25$$

$$70.866\,u_2 = 0.25$$

$$\boxed{u_2 = 0.003\text{ mm}}$$

$$\boxed{u_3 = 0.0192\text{ mm}}$$

4. A composite wall is made up of tow materials as shown in Figure 2. The thermal conductivities of material 1 and 2 are 20 and 30 W/m^{0}C respectively. Hot air at 1,000 ^{0}C is present to the left of wall 1 and the film coefficient at the interface is 25 W/m$^{2.\,0}$C. The right side wall is maintained at 50 ^{0}C

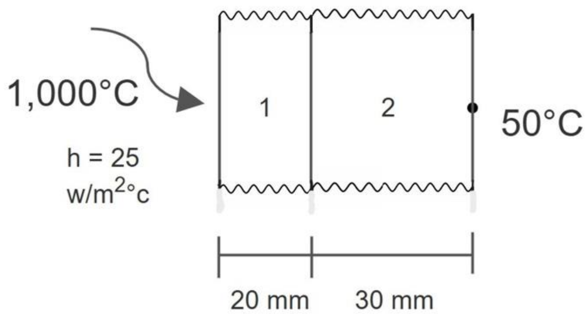

Figure 2

Find the wall temperature at the intersection of the two materials and at the inner wall. What is the heat flux across the composite wall.

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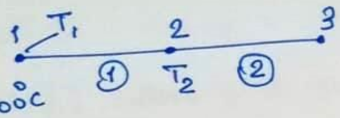

$T_3 = 50°c$

$T_\infty = 1000°c$

$h = 25\ W/m^2°c$

$\boxed{\text{Element ①}}$

Conduction matrix

$$[k]_{conduction} = \frac{kA}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \frac{20(1)}{0.02}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= \begin{bmatrix}1000 & -1000\\ -1000 & 1000\end{bmatrix}$$

At node ① Free end convection

$$[k]_{convection} = \begin{bmatrix}hA & 0\\ 0 & 0\end{bmatrix} = \begin{bmatrix}25 & 0\\ 0 & 0\end{bmatrix}$$

$$[k]_{Thermal} = [k]_{conduction} + [k]_{convection}$$

$$= \begin{bmatrix}1025 & -1000\\ -1000 & 1000\end{bmatrix}$$

$$[q] = \begin{Bmatrix}hAT_\infty\\ 0\end{Bmatrix} = \begin{Bmatrix}25000\\ 0\end{Bmatrix}$$

$\boxed{\text{Element ②}}$

$$[k]_{conduction} = \frac{30(1)}{0.03}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \begin{bmatrix}1000 & -1000\\ -1000 & 1000\end{bmatrix}$$

$$\begin{bmatrix}1025 & -1000 & 0\\ -1000 & 2000 & -1000\\ \cancel{0} & \cancel{-1000} & \cancel{1000}\end{bmatrix}\begin{Bmatrix}T_1\\ T_2\\ \cancel{T_3}\ 50°c\end{Bmatrix} = \begin{Bmatrix}25000\\ 0\\ 0\end{Bmatrix}$$

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	$1025\,T_1 - 1000\,T_2 = 25000$ — (1) $-1000\,T_1 + 2000\,T_2 = 50000$ — (2) Solving (1) and (2) $\boxed{T_1 = 95.23^\circ C}$ $\boxed{T_2 = 72.6^\circ C}$ heat flux across the wall $q = hA\,\Delta T$ $= 25(1)(95.23 - 50)$ $\boxed{q = 1130.75\ W}$
5.	A cantilever beam of span 2 m is loaded with a concentrated load of 20 kN at the free end. The cross section of the beam is square of side 50 mm. The Young's modulus of the material is 200 GPa. Discretize the beam in to two elements and find the global stiffness equation. Also give the boundary conditions for the problem.

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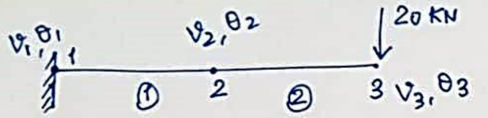

FE Model

$$\frac{EI}{L^3} = \frac{200\times10^9 \times (0.05)^4}{(1)^3 (12)}$$

$$\boxed{\frac{EI}{L^3} = 104166.67 \text{ N/m}}$$

Element ①

$$\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{Bmatrix}$$

Element ②

$$\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} v_2 \\ \theta_2 \\ v_3 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ M_2 \\ F_3 \\ M_3 \end{Bmatrix}$$

Global Stiffness equation

$$104166.67\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 12+12 & -6L+6L & -12 & 6L \\ 6L & 2L^2 & -6L+6L & 4L^2+4L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} \cancel{v_1}\,0 \\ \cancel{\theta_1}\,0 \\ v_2 \\ \theta_2 \\ v_3 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ M_1 \\ \cancel{F_2}\,0 \\ \cancel{M_2}\,0 \\ \cancel{F_3}\,20\text{ KN} \\ \cancel{M_3}\,0 \end{Bmatrix}$$

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