REG.NO.:

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF ADVANCED SCIENCES
CONTINUOUS ASSESSMENT TEST - I
FALL SEMESTER 2025-2026

SLOT: E2+TE2

Programme Name & Branch	**: B. Tech**
Course Code	**: BAMAT101**
Course Name	**: Multivariable Calculus and Differential Equations**
Faculty Name(s)	**: Common question paper for E2+TE2 slot**
Class Number(s)	**: Common to all faculties handling E2+TE2 slot**
Date of Examination	**: 21/08/2025**
Exam Duration	**: 90 minutes** — **Maximum Marks: 50**

General instruction(s):

- Answer All Questions
- M - Max mark; CO – Course Outcome; BL – Blooms Taxonomy Level (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create)
- Course Outcomes (Type the CO statements covered in this question paper. Use the CO number as per the syllabus copy)
  CO-1 Evaluate partial derivatives and solve optimization problems involving several variables with or without constraints.
  CO-2 Solve multiple integrals in cartesian, polar, cylindrical and spherical coordinates.

Q. No	Question	M	CO	BL
1.	Show that the functions $u = x + y + z, v = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ and $w = x^3 + y^3 + z^3 - 3xyz$ are functionally related. Find the relation between them.	10	1	3
2.	Obtain the second order Taylor' series approximation to the function $f(x, y) = xy^2 + y\cos(x - y)$ about the point (2,1).	10	1	1
3.	a. Find and classify the critical points of $f(x, y) = x^3 - 3xy^2$.	3	2	4
	b. A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters earth's atmosphere and its surface begins to heat. After 1 hour, the temperature at a point on the probe's surface is $\theta = 8x^2 + 4yz - 16z + 600$. Find the hottest point on the probe's surface.	7		
4.	Evaluate $\iint_D (7x^2 + 14\,y)\,dA$, where D is the region bounded by $x = 2y^2$ and $x = 8$ in the order given below. (a) Integrate with respect to x first and then y. (b) Integrate with respect to y first and then x.	10	2	1
5.	Evaluate the integral $\int_0^\infty \int_0^x x\, e^{-\frac{x^2}{y}}\, dy\, dx$.	10	2	2

E2 slot ~~zu~~ Scheme

Q1. $u = x+y+z$, $v = x^2+y^2+z^2-2xy-2yz-2zx$,

$w = x^3+y^3+z^3-3xyz$

$$J\left(\frac{u,v,w}{x,y,z}\right) = \begin{vmatrix} 1 & 1 & 1 \\ 2x-2y-2z & 2y-2x-2z & 2z-2y-2x \\ 3x^2-3yz & 3y^2-3xz & 3z^2-3xy \end{vmatrix} \quad — 2$$

$$= 6 \begin{vmatrix} 1 & 1 & 1 \\ x-y-z & y-x-z & z-y-x \\ x^2-yz & y^2-zx & z^2-xy \end{vmatrix} \quad \begin{matrix} C_1 \to C_1 - C_2 \\ C_2 \to C_2 - C_3 \end{matrix}$$

$$= 12 \begin{vmatrix} x-y & y-z \\ (x-y)(x+y+z) & (y-z)(x+y+z) \end{vmatrix} = 0 \quad — 4$$

Hence they are functionally dependent.

Now, $v = (x+y+z)^2 - 4(xy+yz+zx)$

$w = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$\Rightarrow \quad w = \frac{1}{4}u(u^2+3v)$ — 4

Q2

$f_x = y^2 - y\sin(x-y)$	$f_x(2,1) = 1-\sin 1$
$f_y = 2xy + y\sin(x-y) + \cos(x-y)$	$f_y(2,1) = 4+\sin 1+\cos 1$
$f_{xx} = -y\cos(x-y)$	$f_{xx}(2,1) = -\cos 1$
$f_{xy} = 2y - \sin(x-y) + y\cos(x-y)$	$f_{xy}(2,1) = 2-\sin 1+\cos 1$
$f_{yy} = 2x + 2\sin(x-y) - y\cos(x-y)$	$f_{yy}(2,1) = 2+2\sin 1-\cos 1$
5	2

$$f(x,y) \approx f(2,1) + (x-2) f_x(2,1) + (y-1) f_y(2,1) + \frac{1}{2}(x-2)^2 f_{xx}(2,1) + (x-2)(y-1) f_{xy}(2,1) + \frac{1}{2}(y-1)^2 f_{yy}(2,1)$$ — 1

Substituting the values and writing the final answer. — 2

Q3a

$f_x = 3x^2 - 3y^2$, $f_y = -6xy$

$\Rightarrow (0,0)$ is a critical point. — 1

$f_{xx}(0,0) = 0$, $f_{xy}(0,0) = 0$, $f_{yy}(0,0) = 0$

$$\Rightarrow \Delta = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = 0$$ — 1

$\Rightarrow$ Test is inconclusive. — 1

b Here $F(x,y,z) = 8x^2 + 4yz - 16z + 600 + \lambda(4x^2 + y^2 + 4z^2 - 16)$ — 1

$$\left.\begin{aligned} F_x &= 16x + 8\lambda x = 0 \\ F_y &= 4z + 2\lambda y = 0 \\ F_z &= 4y - 16 + 8\lambda z = 0 \end{aligned}\right\} — 2$$

$\Rightarrow$ Hottest points are $\left(\pm\frac{4}{3}, -\frac{4}{3}, -\frac{4}{3}\right)$ — 4

Q4 a) $$I = \int_{-2}^{2}\int_{2y^2}^{8} (7x^2 + 14y)\,dx\,dy$$ — 2

$$= \int_{-2}^{2} \frac{7}{3}(512 - 8y^6) + 14y(8 - 2y^2)\,dy = 4096$$ — 3

b) $I = \int_0^8 \int_{-\sqrt{x/2}}^{\sqrt{x/2}} (7x^2 + 14y)\, dy\, dx$ — 2

$$= \int_0^8 14x^2 \sqrt{\frac{x}{2}}\, dx = 4096 \quad - 3$$

Q5 Changing order of integration — 3

$$I = \int_0^\infty \int_y^\infty x e^{-x^2/y}\, dx\, dy \quad - 3$$

$$= \int_0^\infty \frac{y}{2} e^{-y}\, dy = \frac{1}{2} \quad - 4$$