U/D/TY

Final Assessment Test – April 2025

VIT
Vellore Institute of Technology

Course: IMAT102L - Differential Equations and Transforms

Class NBR(s): 4867 / 4869 / 4870 / 4872 / 6106

Slot: A1+TA1+TAA1

Time: Three Hours

Max. Marks: 100

- KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- DON'T WRITE ANYTHING ON THE QUESTION PAPER

Answer ALL Questions

(10 X 10 = 100 Marks)

1. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{1}{1+e^{-x}}$ by using Method of Variation of Parameters. [10]

2. A condenser of capacity C discharged through an inductance L and resistance R in series and the charge q at time t satisfies the equation $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E$. Given that $L = 0.25$ henries, $R = 12.5\ ohms,\ C = 10^{-2}$ farads, $E = 3\sin 2t$, and that when $t = 0$, charge in the capacitor and current in the circuit are zero. Find the charge q in terms of t by using method of variation of parameters. [10]

3. (i) Form the partial differential equation by eliminating the arbitrary constants a, b and c from $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$ [10]

   (ii) Form the partial differential equation by eliminating the arbitrary functions from $z = \frac{1}{x}[f(x-ay) + g(x+ay)]$.

4. Solve the partial differential equation $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$. [10]

5.a) Find the Laplace transform of $f(t) = \begin{cases} t & 0 \le t < \pi \\ \pi - t & \pi < t \le 2\pi \end{cases}$ where $f(t) = f(t + 2\pi)$ and also draw the function. [10]

OR

5.b) Find the inverse Laplace transform $y(t)$ of the function $Y(s) = \frac{s}{s^4+4a^4}$. [10]

6. Solve $\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial t} = 2x$ where $x > 0, t > 0, u(x,0) = 1, u(0,t) = 1$ by using Laplace Transforms. [10]

7.a) Obtain the Fourier series for the function $f(x) = x\sin x$ in the interval $0 \le x \le 2\pi$ and hence evaluate the value of $\frac{1}{1.3} + \frac{1}{2.4} + \frac{1}{3.5} + \cdots \infty$. [10]

OR

7.b) Obtain the Fourier series for the function $f(x) = \begin{cases} \pi x & 0 \le x < 1 \\ \pi(2-x) & 1 < x \le 2 \end{cases}$. [10]

And hence deduce that the value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots \infty$.

8. Find the Fourier cosine transform of $f(x) = \frac{1}{x^2+1}$ and hence derive Fourier sine transform of $f(x) = \frac{x}{x^2+1}$. [10]

9. If $U(z) = \frac{2z^2+5z+14}{(z-1)^4}$, then find the value of u_2 and u_3. [10]

10. Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ given that $y_0 = y_1 = 0$ using Z –Transforms. [10]

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