8. Find the Fourier transform of $f(x) = \begin{cases} x, & |x| \le a \\ 0, & otherwise \end{cases}$ and hence evaluate [10]
$\int_{-\infty}^{\infty} \left(\frac{\cos x}{x} - \frac{\sin x}{x^2}\right) dx.$

9. Solve the difference equation $x_{n+2} - 8x_{n+1} + 15x_n = 0$ with $x_0 = 3$, $x_1 = 2$ by using Z-transform. [10]

10. (i) Find the Z transform of $v(n) = 3^{n-2}u(n-2)$, here $u(n-2)$ is a Heaviside function. [5]

(ii) Find $Z(x_n * y_n)$ if $x_n = 2^n$ and $y_n = 3^n$ using convolution theorem where $*$ denotes the convolution. [5]

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VIT
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Final Assessment Test – April 2025

Course: IMAT102L - Differential Equations and Transforms

Class NBR(s): 4873 / 4874 / 4875 / 5455

Slot: A2+TA2+TAA2

Time: Three Hours

Max. Marks: 100

- KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- DON'T WRITE ANYTHING ON THE QUESTION PAPER

Answer ALL Questions
(10 X 10 = 100 Marks)

1. Solve the differential equation $xy'' + y' - 9\frac{y}{x} = x\log x,\ x > 0$ by using the method of undetermined co-efficients. [10]

2. Find the steady state current $i(t)$ in an LCR circuit with $R = 20\Omega$, $C = 5 \times 10^{-3}F$ and $L = 0.5H$, which is connected to a source voltage $E(t) = -\frac{1}{40}e^{-20t}$. Assume that the initial charge and current is zero. Formulate the problem into differential equation and solve it by using method of variation of parameter. [10]

3. (i) Form the partial differential equation by elimination of the arbitrary constants 'a' and 'b' from $(x-a)^2 + (y-b)^2 = z^2\cot^2\alpha$. [5]

   (ii) Find the complete solution of the partial differential equation $p(1-q^2) = q(1-z)$. [5]

4. Find the general solution of the partial differential equation $p + 3q = 5z + \tan(y - 3x)$. [10]

5.a) Find the Laplace transform of $f(t) = \begin{cases} t\ , & 0 \le t \le a \\ 2a - t\ , & a \le t \le 2a \end{cases}$ with $f(t + 2a) = f(t)$. [10]

OR

5.b) State convolution theorem of Laplace transforms and hence using it to find the inverse Laplace transform of $\frac{s^4+s^3}{(s^4-s^2)(s^2+4)}$. [10]

6. Solve the differential equation $x\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = xt,\ u(x,0) = 0$ and $u(0,t) = 0$ by using Laplace transform. [10]

7.a) Find the Fourier series of the function $f(x) = \begin{cases} \pi - x\ , & 0 \le x \le \pi \\ 0\ , & \pi \le x \le 2\pi \end{cases}$ with $f(x + 2\pi) = f(x)$. [10]

OR

7.b) Find the half range Fourier sine series of the function $f(x) = 2x - x^2$, $0 < x < \frac{3}{2}$ and hence deduce $\frac{1}{1^2} - \frac{3}{2^2} + \frac{1}{3^2} - \frac{3}{4^2} + \cdots\infty = \frac{3\pi^2}{4}$. [10]