KEY

Course Name:- Differential Equations and Transforms.
Course Code:- BMAT102L, Slot:- C2+TC2+TCC2.

1. $2\frac{d^2y}{dt^2} + \frac{dy}{dt} - y = t+1$

The A.E is $2m^2+m-1=0$

$(2m-1)(m+1)=0$

$m = \frac{1}{2}, -1$

$\therefore y_c = c_1 e^{t/2} + c_2 e^{-t}$

$w = \begin{vmatrix} e^{t/2} & e^{-t} \\ \frac{1}{2}e^{t/2} & -e^{-t} \end{vmatrix} = -\frac{3}{2} e^{-t/2}$ → 4M

$u_1(t) = -e^{-t/2}\left(\frac{2}{3}t - 2\right),\ u_2(x) = -\frac{1}{3} t e^{t}$

$\therefore y(t) = c_1 e^{t/2} + c_2 e^{-t} - t - 2$ → 4M

$y'(t) = \frac{1}{2} c_1 e^{t/2} - c_2 e^{-t} - 1$

I.C :- $c_1 - c_2 - 2 = 1$

$\frac{1}{2}c_1 - c_2 - 1 = 0$

$\therefore c_1 = \frac{8}{3};\ c_2 = \frac{1}{3}$ → 2M

$\therefore y(t) = \frac{8}{3} e^{t/2} + \frac{1}{3} e^{-t} - t - 2$

2. $\frac{d^2v}{dr^2} + \frac{1}{r}\frac{dv}{dr} = \frac{12 \log r}{r^2}$

$\Rightarrow r^2 \frac{d^2v}{dr^2} + r\frac{dv}{dr} = 12 \log r$

Let $r = e^z;\ z = \log r$

$r^2 \frac{d^2v}{dr^2} = \frac{d^2v}{dz^2} - \frac{dv}{dz}$

$r\frac{dv}{dr} = \frac{dv}{dz}$

$\frac{d^2v}{dz^2} - \cancel{\frac{dv}{dz}} + \cancel{\frac{dv}{dz}} = 12z$

$\therefore \frac{d^2v}{dz^2} = 12z$ (or) direct Integration

$v_c = c_1 + c_2 z = c_1 + c_2 \log r$

$v_p = 2z^3 = 2(\log r)^3$

$\therefore v = c_1 + c_2 \log r + 2(\log r)^3$

3. $\frac{1}{2}x'' + \frac{1}{2}x' + 6x = 10\cos 3t$

$x(0) = -2\,;\ x'(0) = 0$

$x_c(t) = e^{-t/2}\left(c_1 \cos\frac{\sqrt{47}}{2}t + c_2 \sin\frac{\sqrt{47}}{2}t\right)$

$x_p(t) = \frac{10}{3}(\cos 3t + \sin 3t)$

$\therefore$ The equation of motion is

$x(t) = e^{-t/2}\left(-\frac{4}{3}\cos\frac{\sqrt{47}}{2}t - \frac{64}{3\sqrt{47}}\sin\frac{\sqrt{47}}{2}t\right) + \frac{10}{3}(\cos 3t + \sin 3t).$

4. $x(z^2 - y^2)\frac{\partial z}{\partial x} + y(x^2 - z^2)\frac{\partial z}{\partial y} = z(y^2 - x^2)$

The auxiliary simultaneous equations are

$$\frac{dx}{x(z^2-y^2)} = \frac{dy}{y(x^2-z^2)} = \frac{dz}{z(y^2-x^2)}$$

using multipliers x, y, z we get

$$\frac{x\,dx + y\,dy + z\,dz}{x^2(z^2-y^2) + y^2(x^2-z^2) + z^2(y^2-x^2)} = \frac{x\,dx + dy + z\,dz}{0}$$

$\therefore x\,dx + y\,dy + z\,dz = 0$

$x^2 + y^2 + z^2 = c_1$

Again $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$, we get

$$\frac{\frac{dx}{x}}{z^2-y^2} = \frac{\frac{dy}{y}}{x^2-z^2} = \frac{\frac{dz}{z}}{y^2-x^2} = \frac{\frac{dx}{x}+\frac{dy}{y}+\frac{dz}{z}}{z^2-y^2+x^2-z^2+y^2-x^2} = \frac{\frac{dx}{x}+\frac{dy}{y}+\frac{dz}{z}}{0}$$

$\therefore \log x + \log y + \log z = \log c_2$

$\therefore xyz = c_2$

$\therefore \phi(x^2+y^2+z^2, xyz) = 0$

5. $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u\ ;\ u(0,y) = e^{-5y}$

Let the solution be $u(x,y) = X(x)Y(y)$

$4X'Y + XY' = 3XY$

$\Rightarrow 4X'Y = (3Y - Y')X$

$\Rightarrow \frac{4X'}{X} = \frac{3Y - Y'}{Y} = k$ (say)

$\frac{X'}{X} = \frac{k}{4}\ ;\ \frac{Y'}{Y} = 3 - k$

$\therefore X = c_1 e^{\frac{kx}{4}}\ ;\ Y = c_2 e^{(3-k)y}$

$\therefore u(x,y) = c_1 c_2 e^{\frac{kx}{4}} e^{(3-k)y}$

But $u(0,y) = e^{-5y}$

$c_1 c_2 = 1\ ;\ k = 8$

$\boxed{\therefore u(x,y) = e^{2x-5y}.}$

REG.NO.:

VIT
Vellore Institute of Technology

SCHOOL OF ADVANCED SCIENCES
CONTINUOUS ASSESSMENT TEST - I
WINTER SEMESTER 2024-2025

SLOT: C2+TC2+TCC2

Programme Name & Branch	: B. Tech., All Branches
Course Code and Course Name	: BMAT102L, Differential Equations and Transforms
Class Number(s)	: Common.
Date of Examination	: 29-01-2025
Exam Duration	: 90 minutes

Maximum Marks: 50

General instruction(s): Answer All Questions

Q. No	Question	M	CO	BL
1.	Find the solution of the differential equation $2\frac{d^2y}{dt^2}+\frac{dy}{dt}-y=t+1$ by using variation of parameters subject to the initial conditions $y(0)=1, y'(0)=0$.	10	1	2
2.	Solve the differential equation: $\frac{d^2v}{dr^2}+\frac{1}{r}\frac{dv}{dr}=\frac{12\log r}{r^2}$.	10	1	2
3.	A mass weight 16 pounds stretches a spring $\frac{8}{3}$ feet. The mass is initially released from rest from a point 2 feet below the equilibrium position, and the subsequent motion takes place in a medium that offers a damping force numerically equal to one-half the instantaneous velocity. Find the equation of motion if the mass is driven by an external force equal to $f(t)=10\cos(3t)$.	10	1	3
4	Find the general solution of $x(z^2-y^2)p+y(x^2-z^2)q=z(y^2-x^2)$.	10	2	2
5.	Using the separation of variables, solve the equation $4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u$ and $u(0,y)=e^{-5y}$.	10	2	2

****