Final Assessment Test – April 2025

VIT® Vellore Institute of Technology

Course: BMAT102L - Differential Equations and Transforms

Class NBR(s): 1094 / 1095 / 1096 / 1097 / 1098 / 1099 / 1100 / 1101 / 1102 / 1103 / 1104 / 1105 / 1106 / 1107 / 1108 / 1109 / 1110 / 1111 / 1112 / 1113 / 1114 / 1115 / 1116 / 1117 / 1118 / 6108

Slot: A1+TA1+TAA1

Time: Three Hours

Max. Marks: 100

- **KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE**
- **DON'T WRITE ANYTHING ON THE QUESTION PAPER**

Answer ALL Questions

(10 X 10 = 100 Marks)

1. Solve $(D^2 - 4D + 3)y = e^x \cos 2x$, by the method of undetermined coefficients. [10]

2. Solve $(x^2D^2 - xD + 1)y = x \log x$, by the method of variation of parameters. [10]

3. (i) Find the complete and singular solutions of $px - z = 2\sqrt{pq} - yq$. [5]

   (ii) Form the partial differential equation by eliminating the arbitrary function f from $f(ax + by + cz,\ x^2 + y^2 + z^2) = 0$. [5]

4. (i) Solve: $x^2p - y^2q = z(x - y)$. [5]

   (ii) Solve: $9(zp^2 + q^2) = 4$. [5]

5. Find the Laplace transform of the periodic function $f(t)$ of period $2a$, where [10]
$$f(t) = \begin{cases} t, & ;\ \text{in } 0 < t < a \\ 2a - t; & \text{in } a \le t < 2a \end{cases}.$$

6. Using the convolution theorem, find $L^{-1}\left(\frac{1}{(s^2+1)\,(s^2+4)}\right)$. [10]

7.a) Using Laplace transform, solve the differential equation [10]
$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} - 4y = g(t), \quad y(0) = 0,\ y'(0) = 0. \text{ where}$$
$$g(t) = \begin{cases} e^t, & 0 < t < 2 \\ 0, & t > 2 \end{cases}.$$

OR

7.b) Using Laplace transform, Solve the partial differential equation [10]
$$u_x = 2u_t + u,\ \ u(x,0) = 6\,e^{-3x}, \text{ which is bounded for all } x > 0,\ \ t > 0.$$

8.a) Obtain the Fourier series of period 2π given by [10]

$$f(x) = \begin{cases} -\pi x - x^2; & -\pi < x < 0 \\ \pi x - x^2 ; & 0 < x < \pi \end{cases}$$

Deduce the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

OR

8.b) Find the Fourier series of the function $f(x) = x^2$, in $(-1, 1)$. [10]

Deduce the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^4}$.

9. Find the Fourier transform of $f(x) = \begin{cases} 1, & |x| < a \\ 0, & x > a \end{cases}$. Hence evaluate the following [10]

(i) $\int_0^{\infty} \frac{\sin t}{t} dt$

(ii) $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^2 dt$.

10. Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, $y(0) = 0 = y(1)$. Using Z-transforms. [10]

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