


VIT

Vellore Institute of Technology

Final Assessment Test - April 2025

 Course: **BMAT102L - Differential Equations and Transforms**

 Class NBR(s): **1119 / 1120 / 1121 / 1122 / 1123 / 1124 /**
1125 / 1126 / 1127 / 1128 / 1129 / 1130

 Slot: **A2+TA2+TAA2**

 Time: **Three Hours**

 Max. Marks: **100**

- **KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE!**
- **DON'T WRITE ANYTHING ON THE QUESTION PAPER**

 Answer ALL Questions

(10 X 10 = 100 Marks)

1. Solve: $y'' + 3y' + 2y = 2e^x$, using the method of variation of parameters. [10]
2. Solve: $2x^2y'' + 3xy' - 3y = x^3$ using the method of undetermined coefficients. [10]
3. (i) Form the partial differential equation by eliminating arbitrary functions from $z = f(x+at) + g(x-at)$ [5]
- (ii) Find the general solution of the equation [5]

$$\frac{dx}{x(y^2 - z^2)} = \frac{dy}{-y(z^2 + x^2)} = \frac{dz}{z(x^2 + y^2)}$$
 using method of multipliers.
4. Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where [10]

$$u(x, 0) = 6e^{-3x}.$$
5. (i) Find Laplace transform of $f(t) = (\cos t + \sin t)^2$ [5]
- (ii) Find the Laplace transform of the square wave given by [5]

$$f(t) = \begin{cases} k & , 0 \leq t < a, \\ -k & , a \leq t < 2a, \end{cases} \text{ and } f(t+2a) = f(t).$$
6. Find the Inverse Laplace transform of the function $\frac{s}{(s^2+4)(s^2+9)}$ using [10] convolution theorem.
- 7.a) Solve the initial value problem $y'' + 2y' + 10y = 3\delta(t-3)$, $y(0) = 0, y'(0) = 0$ using [10] Laplace transforms.
- OR** [10]
- 7.b) Solve by the Laplace transforms method, of the first order initial boundary value [10] problem $\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial t} = 4xt$, $u(x, 0) = 0, u(0, t) = 0$.
- 8.a) Find the Fourier series expansion of the periodic function [10]

$$f(x) = \begin{cases} \pi + x, & -\pi < x < 0 \\ 0, & 0 \leq x < \pi \end{cases} \text{ with period } 2\pi.$$
- OR** [10]
- 8.b) Find the half range Fourier cosine series for the function $f(x) = (x-1)^2$ in the [10] interval $0 < x < 1$. Hence find $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

9. Find the Fourier transform of the function $f(x) = \begin{cases} 0, & x < 0 \\ 1, & 0 \leq x \leq 1 \\ 0, & x > 1 \end{cases}$. Hence, evaluate [10]

$$\int_0^{\infty} \frac{\sin\left(\frac{x}{2}\right)}{x} dx.$$

10. Solve the difference equation $y_{n+2} - 2y_{n+1} + y_n = n$, $y_0 = 1$, $y_1 = 1$ using Z transforms. [10]

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