



- **KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE**
- **DON'T WRITE ANYTHING ON THE QUESTION PAPER**

Answer ALL Questions
(10 X 10 = 100 Marks)

1. Obtain the solution of the differential equation $x^2y'' + 10xy' + 8y = x^2$ using the method of variation of parameters. [10]
 2. An LCR circuit connected in series has $R = 16$ ohms, $C = 0.02$ farad, $L = 2$ henry and an applied voltage $E = 100 \sin 3t$ volts. At $t = 0$, the charge on the capacitor and the current in the circuit are zero. Find the charge on the capacitor at any time t . [10]
 3. i) Form the partial differential equation by eliminating the arbitrary function f from the relation $z = xy + f(x^2 + y^2)$. [5]
ii) Solve $px^2 + qy^2 = z(x + y)$, where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$. [5]
 4. Solve $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial t} = 0$ with $u(x, 0) = 4e^{-x}$ using the method of separation of variables. [10]
 5. i) Find $L \left\{ e^{-2t} \int_0^t \left(\frac{\sin u}{u} \right) du \right\}$. [5]
ii) Find the Laplace transform of the periodic function $f(t) = \begin{cases} 1 & \text{if } 0 < t < 1 \\ t & \text{if } 1 < t < 2 \end{cases}$ with period 2. [5]
 6. Find the inverse Laplace transform of $\frac{1}{s(s^2 + 2s + 2)}$ using the partial fractions method. [10]
 - 7.a) Find the solution of $y'' - y' - 2y = \delta(t - 1)$ with $y(0) = 0$, $y'(0) = 1$, using the Laplace transform. [10]
- OR**
- 7.b) Obtain the solution of $u_x + u_t + u = 0$ with $u(0, t) = 0 (t > 0)$, $u(x, 0) = \sin x (x > 0)$ using the Laplace transform.

8.a) Obtain the Fourier series for $f(x) = x - x^2$ in the interval $[-\pi, \pi]$, and hence [10]

find the sum of the series $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$

OR

8.b) Find the half-range sine series of $f(x) = \begin{cases} x & \text{if } 0 < x < \frac{\pi}{2} \\ \pi - x & \text{if } \frac{\pi}{2} < x < \pi \end{cases}$, and hence [10]

find the sum of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

9. Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{if } |x| \leq 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ and hence evaluate [10]

$\int_0^{\infty} \left(\frac{\sin x}{x} \right)^2 dx$ using the Parseval's identity.

10. Solve $u_{n+2} - 4u_{n+1} + 4u_n = 1^n$ with $u_0 = 0$, $u_1 = 0$ using the Z-Transform. [10]

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