VIT
Vellore Institute of Technology

Final Assessment Test – April 2025

Course: BMAT102L - Differential Equations and Transforms

Class NBR(s): 1048 / 1049 / 1051 / 1052 / 1053 / 1054 / 1055 / 1056 / 1057 / 1058 / 1059 / 1060 / 1061 / 1062 / 1063 / 1064 / 1065 / 1066 / 1067 / 1068 / 1069 / 1070 / 3640

Slot: C1+TC1+TCC1

Time: Three Hours

Max. Marks: 100

- KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- DON'T WRITE ANYTHING ON THE QUESTION PAPER

Answer ALL Questions

(10 X 10 = 100 Marks)

1. Solve $t^2\frac{d^2x}{dt^2}t\frac{dx}{dt}+9x=\log t$ by the method of variation of parameters. **[10]**

2. Use the method of undetermined coefficients, to solve the differential equation **[10]**

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=e^{2x}+cos2x.$$

3. (i) Obtain the partial differential equation by eliminating the arbitrary function from $\phi(x^2+y^2+z^2,z^2-2xy)=0$. **[5]**

(ii) Solve $q^2=z^2p^2(1-p^2)$, where $p=\frac{\partial z}{\partial x},q=\frac{\partial z}{\partial y}$. **[5]**

4. Solve $4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u,\mathrm{u}(0,\mathrm{y})=3\mathrm{e}^{-y}-e^{-5y}$ using the method of separation of variables. **[10]**

5. (i) Evaluate the improper integral $\int_0^\infty\frac{\cos 2t-\cos 3t}{t}dt$ using Laplace transform. **[10]**

(ii) Find the Laplace transform of a periodic function $f(t)=\begin{cases}1, & 0<t<1\\ t, & 1<t<2\end{cases}$ with period 2.

6. Using the Convolution theorem, find $L^{-1}\left[\frac{s}{s^4+37s^2+36)}\right]$. **[10]**

7.a) Solve the following differential equation using Laplace transform. **[10]**

$$\frac{d^2x}{dt^2}+4x=H(t-\pi)+H(t-2\pi),\ \ x(0)=0=x'(0).$$

OR

7.b) Obtain the solution of $u_x+u_t+u=0$ with **[10]**

$u(0,t)=0\ (t>0),u(x,0)=\cot x\ (x>0)$ using Laplace Transform.

8.a) Find the Fourier series for the function $f(x) = x \sin x,\ 0 < x < 2\pi$. [10]

OR

8.b) Find the half range Fourier cosine series of $f(x) = x(2-x)$ in $0 \le x \le 3$, and hence find the sum $\frac{1}{1^2} - \frac{3}{2^2} + \frac{1}{3^2} - \frac{3}{4^2} + \cdots$ [10]

9. Find the sine transform of $f(x) = \frac{e^{-3x}}{x}$. [10]

10. Find the response of the system $x_{n+2} - 8x_{n+1} + 15x_n = 1$, with $x_0 = 0, x_1 = 0$ using Z-transform. [10]

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