BH/D/TY

Final Assessment Test – April 2025

VIT
Vellore Institute of Technology

Course: **BMAT102L - Differential Equations and Transforms**

Class NBR(s): **1071 / 1072 / 1073 / 1074 / 1075 / 1077 / 1078 / 1079 / 1080 / 1081 / 1082 / 1083 / 1084 / 1085 / 1086 / 1087 / 1088 / 1089 / 1090 / 1091 / 1092 / 1093**

Slot: **C2+TC2+TCC2**

Time: **Three Hours** Max. Marks: **100**

- **KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE**
- **DON'T WRITE ANYTHING ON THE QUESTION PAPER**

Answer ALL Questions
(10 X 10 = 100 Marks)

1. Solve the differential equation by the method of variation of parameters. **[10]**
$(x^2D^2 - 4xD + 6)y = \sin(\log x)$, where $D = \frac{d}{dx}$.

2. The LCR- circuit equation, charge $q(t)$ on a plate of the condenser is given by $\frac{d^2q}{dt^2} + 2\frac{dq}{dt} + q = 2\sin 2t$. Find the charge q at time t, if initially $q = 0$ and $i = 0$ **[10]**

3. (i) Form the partial differential equation by eliminating the arbitrary function from the relation $z = x^2 + 2f\left(\frac{1}{y} + \log x\right)$.
(ii) Solve: $p^2 + pq = z^2$. **[5]**

4. Find the general solution of the partial differential equation **[10]**
$(x + 2z)p + (4xz - y)q = 2x^2 + y$.

5. Find the Laplace transform of the function $f(t) = \begin{cases} \sin t, & 0 \le t \le \pi \\ 0, & \pi \le t \le 2\pi \end{cases}$ and $f(t + 2\pi) = f(t)$. **[10]**

6. Using convolution theorem, find $L^{-1}\left\{\frac{10}{(s+1)(s^2+4)}\right\}$. **[10]**

7.a) Using Laplace transform, solve the differential equation: **[10]**
$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 15y = 6\delta(t - 9)$ with $y(0) = -5$, $y'(0) = 7$.

OR

7.b) Using Laplace transform, solve the partial differential equation **[10]**
$u_x + xu_t = 0$, $x > 0, t > 0$ with $u(x, 0) = 0$, $u(0, t) = t$.

8.a) Find the Fourier series of period 2π of the function $f(x) = \frac{1}{2}(\pi - x)$ in the interval $0 < x < 2\pi$. **[10]**

OR

8.b) Find the half-range cosine series of the function $f(x) = x$, $0 < x < l$ and hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^4}$. **[10]**

9. Find the Fourier transform of $f(x) = \begin{cases} 1-|x|, & |x|<1 \\ 0, & |x|>1 \end{cases}$. Hence evaluate $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^4 dt$. [10]

10. Solve the difference equation: $y_{n+2} - 5y_{n+1} + 6y_n = (-1)^n$ given that $y_0 = 0;\ y_1 = 0$ by using Z transforms. [10]

⇔⇔⇔ BH/D/TY ⇔⇔⇔