



VIT

Vellore Institute of Technology

Final Assessment Test – April 2025

Course: **BMAT102L - Differential Equations and Transforms**

Class NBR(s): **1138 / 1139 / 1140 / 1141 / 1142**

Slot: **D2+TD2+TDD2**

Time: **Three Hours**

Max. Marks: **100**

- **KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE**
- **DON'T WRITE ANYTHING ON THE QUESTION PAPER**

Answer **ALL** Questions

(10 X 10 = 100 Marks)

1. Solve $(D^2 + 4)y = 4 \tan 2x$ by using the method of variation of parameters. [10]
2. In an LCR circuit the impressed voltage is $400 \cos 250t$ volts. Find the current in the circuit when $t = 0.001$ second, if $L = 1$ henry, $R = 400$ ohms and $C = 0.16 \times 10^{-4}$ farad and the initial charge and the current are zero. [10]
3. (i) Solve $z = px + qy + 2\sqrt{pq}$ [5]
 (ii) Solve $(3z - 4y) \frac{\partial z}{\partial x} + (4x - 2z) \frac{\partial z}{\partial y} = 2y - 3x$. [5]
4. Solve $2 \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$, with $z(x, 0) = 5e^{-x}$, by the method of separation of variables. [10]
5. Find the Laplace transform of the function $f(t) = \begin{cases} 1; & 0 < t < \frac{a}{2} \\ -1; & \frac{a}{2} < t < a \end{cases}$ with $f(t+a) = f(t)$. [10]
6. Find $L^{-1} \left[\frac{s^2}{(s^2+2^2)(s^2+3^2)} \right]$, using the convolution theorem. [10]
- 7.a) Solve $x''(t) + 4x'(t) + 4x(t) = \delta(t-1)$ with $x(0) = x'(0) = 0$, using Laplace transform technique. [10]

OR

- 7.b) Using the Laplace transform, solve $\frac{\partial u}{\partial x} + 2x \frac{\partial u}{\partial t} = 2x$, $x > 0$, $t > 0$, with boundary and initial conditions $u(x, 0) = 1$, $u(0, t) = 1$. [10]
- 8.a) Find the Fourier series expansion of $f(x) = \begin{cases} x & \text{in } 0 \leq x \leq \pi \\ 2\pi - x & \text{in } \pi \leq x \leq 2\pi \end{cases}$ in $[0, 2\pi]$. [10]

OR

- 8.b) Find the half range sine series for the function $f(x) = x(\pi - x)$ in the range $(0, \pi)$, also find the sum of the series $1 + \frac{1}{3^3} + \frac{1}{5^3} + \dots + \frac{1}{n^3}$. [10]
9. Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$. Also prove that $\int_0^\infty \frac{s \sin s - s \cos s}{s^3} \cdot \cos\left(\frac{s}{2}\right) ds = \frac{3\pi}{16}$. [10]
10. Using the Z-transform, solve $y_{n+2} + 4y_{n+1} + 3y_n = 3^n$ with $y_0 = y_1 = 0$. [10]