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QP and Key

SCHOOL OF ADVANCED SCIENCES
DEPARTMENT OF MATHEMATICS
FALL SEMESTER – 2025∼2026
CONTINUOUS ASSESSMENT TEST – I **SLOT: C2+TC2+TCC2**

Programme Name & Branch	:	B.Tech. (All Branches)
Course Code & Course Name	:	BMAT205L – Discrete Mathematics and Graph Theory
Class Number(s)	:	Common Slot QP
Faculty Name(s)	:	Common Slot QP
Date of Examination	:	19-08-2025
Exam Duration	:	90 Minutes **Maximum Marks** : 50

General Instruction(s): Answer All Questions.

1. Show that the following statements constitute a valid arguement:

   “*If you send me an e-mail message, then I will finish writing the program.*
   If you do not send me an e-mail message, then I will go to sleep early.
   If I go to sleep early, then I will be feeling refreshed.
   Therefore, if I do not finish writing the program, then I will be feeling refreshed. ”

   Answer(s):
   Notation:
   P : You send me an e-mail message.
   Q : I will finish writing the program.
   R : I will go to sleep early.
   S : I will be feeling refreshed.

   Symbolization: $(P \longrightarrow Q) \wedge (\rceil P \longrightarrow R) \wedge (R \to S) \Longrightarrow (\rceil Q \longrightarrow S)$

   Derivation:

Step No.	Statement	Rule / Formula / Previous Steps Used
1.	$(\rceil P \longrightarrow R)$	Rule P
2.	$(R \to S)$	Rule P
3.	$(\rceil P \longrightarrow S)$	Rule T / Hypothetical Syllogism / 1 & 2
4.	$(\rceil S \longrightarrow \rceil(\rceil P))$	Rule T / Contrapositive Equivalence / 3
5.	$(\rceil S \longrightarrow P)$	Rule T / Negation Law / 4
6.	$(P \longrightarrow Q)$	Rule P
7.	$(\rceil S \longrightarrow Q)$	Rule T / Hypothetical Syllogism / 5 & 6
8.	$(\rceil Q \longrightarrow \rceil(\rceil S))$	Rule T / Contrapositive Equivalence / 7
9.	$(\rceil Q \longrightarrow S)$	Rule T / Negation Law / 8

   Since the conclusion logically follows from the premises, the given argument is **valid**.

   [10 M] [CO: 1] [BL: 2]

2. Obtain the principal disjunctive and principal conjunctive normal forms of the following statement:

$$\Big(P \to (Q \wedge R)\Big) \to \Big(\rceil P \to (\rceil Q \wedge \rceil R)\Big).$$

Answer(s):

Let $S = \Big(P \to (Q \wedge R)\Big) \to \Big(\rceil P \to (\rceil Q \wedge \rceil R)\Big)$.

Truth Table:

P	Q	R	$(Q \wedge R)$	$\Big(P \to (Q \wedge R)\Big)$	$\rceil P$	$(\rceil Q \wedge \rceil R)$	$\Big(\rceil P \to (\rceil Q \wedge \rceil R)\Big)$	S
T	T	T	T	T	F	F	T	T
T	T	F	F	F	F	F	T	T
T	F	T	F	F	F	F	T	T
T	F	F	F	F	F	T	T	T
F	T	T	T	T	T	F	F	F
F	T	F	F	T	T	F	F	F
F	F	T	F	T	T	F	F	F
F	F	F	F	T	T	T	T	T

Principal Disjunctive Normal Form (PDNF):

$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \rceil R) \vee (P \wedge \rceil Q \wedge R) \vee (P \wedge \rceil Q \wedge \rceil R) \vee (\rceil P \wedge \rceil Q \wedge \rceil R)$.

Principal Conjunctive Normal Form (PCNF):

$(\rceil P \vee \rceil Q \vee \rceil R) \wedge (\rceil P \vee \rceil Q \vee R) \wedge (\rceil P \vee Q \vee \rceil R)$.

[10 M] [CO: 1] [BL: 1]

3. Show that the conclusion $\Big(Q(y) \wedge (\exists x)(P(x) \wedge R(x))\Big)$ can be derived from $(\forall x)\Big(P(x) \longrightarrow (Q(y) \wedge R(x))\Big)$ and $(\exists x)(P(x))$.

Answer(s):

Derivation:

Step No.	Statement/Predicate	Rule / Formula / Previous Steps Used
1.	$(\exists x)(P(x))$	Rule P
2.	$P(a)$	Rule ES / x is fixed by a / 1
3.	$(\forall x)\Big(P(x) \longrightarrow (Q(y) \wedge R(x))\Big)$	Rule P
4.	$\Big(P(a) \longrightarrow (Q(y) \wedge R(a))\Big)$	Rule US / x is fixed by a / 3
5.	$(Q(y) \wedge R(a))$	Rule T / Modus Ponens / 2 & 4
6.	$Q(y)$	Rule T / Simplification / 5
7.	$R(a)$	Rule T / Simplification / 5
8.	$(P(a) \wedge R(a))$	Rule T / $P, Q \Longrightarrow (P \wedge Q)$ / 2 & 7
9.	$(\exists x)(P(x) \wedge R(x))$	Rule EG / 8
10.	$\Big(Q(y) \wedge (\exists x)(P(x) \wedge R(x))\Big)$	Rule T / $P(x), Q(x) \Longrightarrow (P(x) \wedge Q(x))$ / 6 & 9

This completes the required proof.

[10 M] [CO: 1] [BL: 3]

4. Show that $G = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \neq 0 \text{ and } a \in \mathbb{R} \right\}$ is a group under the matrix multiplication. Is it an Abelian group?

Answer(s):
For proving the given statement.

(*i*). Closure Property.

$$\begin{pmatrix} a & a \\ a & a \end{pmatrix} + \begin{pmatrix} b & b \\ b & b \end{pmatrix} = \begin{pmatrix} 2ab & 2ab \\ 2ab & 2ab \end{pmatrix} \in G.$$

(*ii*). Associative Property.

$$\left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} + \begin{pmatrix} b & b \\ b & b \end{pmatrix} \right\} + \begin{pmatrix} c & c \\ c & c \end{pmatrix} = \begin{pmatrix} a & a \\ a & a \end{pmatrix} + \left\{ \begin{pmatrix} b & b \\ b & b \end{pmatrix} + \begin{pmatrix} c & c \\ c & c \end{pmatrix} \right\}.$$

(*iii*). Identity Element: $e = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

(*iv*). Inverse Element of $\begin{pmatrix} a & a \\ a & a \end{pmatrix} = \begin{pmatrix} \frac{1}{4a} & \frac{1}{4a} \\ \frac{1}{4a} & \frac{1}{4a} \end{pmatrix}$.

(*v*). Commutative Property.

$$\begin{pmatrix} a & a \\ a & a \end{pmatrix} + \begin{pmatrix} b & b \\ b & b \end{pmatrix} = \begin{pmatrix} b & b \\ b & b \end{pmatrix} + \begin{pmatrix} a & a \\ a & a \end{pmatrix}$$

Hence, the given group is Abelian.

[10 M] [CO: 2] [BL: 3]

5. Show that $H = \{[0], [2], [4]\}$ is a subgroup of a group $(\mathbb{Z}_6, +_6)$. Obtain all the distinct left cosets of H in $\mathbb{Z}_6$ and hence compute the index of H in Z_6.

Answer(s):

***H* is a Subgroup:**

Cayley Table for $(H, +_6)$:

$+_6$	0	2	4
0	[0]	[2]	[4]
2	[2]	[4]	[0]
4	[4]	[0]	[2]

The closure property is satisfied in $(H, +_6)$.

Identity Element: $e = [0] \in H$.

Existence of Inverses:

$$[0]^{-1} = [0] \in H$$
$$[2]^{-1} = [4] \in H$$
$$[4]^{-1} = [2] \in H$$

Hence, $H = \{[0], [2], [4]\}$ is a subgroup of a group $(\mathbb{Z}_6, +_6)$.

Distinct Left Cosets of H in $\mathbb{Z}_6$:
$0 +_6 H = 2 +_6 H = 4 +_6 H = \{[0], [2], [4]\}$
$1 +_6 H = 2 +_6 H = 5 +_6 H = \{[1], [3], [5]\}$

Index of H in Z_6:
Index $[Z_6 : H] = \dfrac{O(G)}{O(H)} = 6/3 = 2$.

[10 M] [CO: 2] [BL: 2]

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