REG. NO.:

SLOT: C1+TC1+TCC1

SCHOOL OF ADVANCED SCIENCES
DEPARTMENT OF MATHEMATICS
CONTINUOUS ASSESSMENT TEST – II
FALL SEMESTER – 2025∼2026

Programme Name & Branch	:	**B.Tech. (All Branches)**
Course Code & Course Name	:	**BMAT205L & Discrete Mathematics & Graph Theory**
Faculty Name(s) & Class Number(s)	:	**Common Question Paper**
Date of Examination and Session	:	**07-10-2025 & AN Session**
Exam Duration	:	**90 Minutes** **Maximum Marks : 50 M**

General Instruction(s):

- Answer All Questions.
- M – Max. Marks; CO – Course Outcome; BL – Blooms Taxonomy Level (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create).
- Course Outcomes:
  CO-2: Use algebraic structures in applications, CO-3: Counting techniques in engineering problems, CO-4: Use lattice and Boolean algebra properties in Digital circuits.
- Students are permitted to carry any number of text books and hand written class note books.

Q. No.	Questions	M	CO	BL
1.	Find the code words generated by the parity check matrix $H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$, when the encoding function is $e : B^3 \longrightarrow B^6$. Also find how many errors e will detect? and how many errors it will correct?	10	2	3
2.	A database assigns unique **record IDs** numbered from 1 to 5000. For system optimization, IDs that are divisible by 3, 5, or 7 are reserved for internal indexing and cannot be given to user records. How many valid record IDs can be assigned to users for internal indexing?	10	3	2

P.T.O

Q. No.	Questions	M	CO	BL
3.	(i). How many different 3 –digit numbers can be formed using the digits 1, 2, 3, 4, 5 without repetition? (ii). In how many ways can the letters of the word **MATH** be arranged? (iii). From 7 men, a committee of 3 is to be formed. In how many ways can this be done? (iv). How many different 4 – letter words can be formed using the letters of the word **BOOK**? (v). In how many ways can 5 different books be arranged on a shelf?	10	3	1
4.	A city decides to plant trees along a highway. In the first year, 50 trees are planted. In the second year, 120 trees are planted. From the third year onwards, the number of trees planted in a year is equal to the sum of trees planted in the previous year and twice the number of trees planted two years earlier. 1. Formulate a recurrence relation for the number of trees planted in year n. 2. Find the number of trees planted in the n^{th} year. 3. Compute the total number of trees planted by the end of the 4th year. .	10	3	3
5.	(1). Let $X_1 = \left\{S_0, S_1, S_2, \cdots, S_7\right\}$ where $S_0 = \{a,b,c,d,e,f\}$, $S_1 = \{a,b,c,d,e\}$, $S_2 = \{a,b,c,e,f\}$, $S_3 = \{a,b,c,e\}$, $S_4 = \{a,b,c\}$, $S_5 = \{a,b\}$, $S_6 = \{a,c\}$ and $S_7 = \{a\}$. (i). Verify whether $(X_1, \subseteq)$ is a partially ordered set or not. (ii). If so, draw a Hasse diagram for the same. (2). Consider the set $X_2 = \{2,3,6,12,24,36\}$ and let for $a, b \in X_2$, "$a \leq b$" be the relation "a divides b" *(i)*. Verify whether X_2 is a Poset or not. *(ii)*. Draw the Hasse diagram of $(X_2, \leq)$. *(iii)*. Check $(X_2, \leq)$ is lattice or not.	10	4	3

****

REG. NO.:

SLOT: C1+TC1+TCC1

SCHOOL OF ADVANCED SCIENCES
DEPARTMENT OF MATHEMATICS
CONTINUOUS ASSESSMENT TEST – II - Answer Key
FALL SEMESTER – 2025∼2026

Programme Name & Branch	:	**B.Tech. (All Branches)**
Course Code & Course Name	:	**BMAT205L & Discrete Mathematics & Graph Theory**
Faculty Name(s) & Class Number(s)	:	**Common Question Paper**
Date of Examination and Session	:	**07-10-2025 & AN Session**
Exam Duration	:	**90 Minutes** **Maximum Marks : 50 M**

General Instruction(s):

- Answer All Questions.
- M – Max. Marks; CO – Course Outcome; BL – Blooms Taxonomy Level (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create).
- Course Outcomes:
  CO-2: Use algebraic structures in applications, CO-3: Counting techniques in engineering problems, CO-4: Use lattice and Boolean algebra properties in Digital circuits.
- Students are permitted to carry any number of text books and hand written class note books.

<table>
<tr><th>Q. No.</th><th>Questions</th><th>M</th><th>CO</th><th>BL</th></tr>
<tr><td>1.</td><td>

Find the code words generated by the parity check matrix $H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$, when the encoding function is $e : B^3 \longrightarrow B^6$. Also find how many errors e will detect? and how many errors it will correct?

Solution:

5. Minimum distance $d_{\min}$

Compute Hamming weights of nonzero codewords (count of 1s):

- 001111: weight 4
- 010101: weight 3
- 011010: weight 3
- 100110: weight 3
- 101001: weight 3
- 110011: weight 4
- 111100: weight 4

Smallest nonzero weight is 3. Hence the minimum distance $d_{\min} = 3$.

The code words are:

$e(000) = 000000$, $e(001) = 001011$, $e(010) = 010101$, $e(100) = 100110$;

$e(011) = 011110$, $e(101) = 101101$, $e(110) = 110011$, $e(111) = 111000$.

- **Theorem: Suppose that e is an (m, n) encoding function and d is a maximum likelihood decoding function associated with e. Then (e, d) can correct k or fewer errors if and only if the minimum distance is at least $2k + 1$.**

Correct: $2k + 1 = 3$ **which implies** $k = 1$. e **will correct 1 or fewer error.**

Theorem 2 An (m, n) encoding function $e: B^m \rightarrow B^n$ can detect k or fewer errors if and if its minimum distance is at least $k + 1$.

Detect: $k + 1 = 3$ **which implies** $k = 2$. e **will detect 2 or fewer errors.**

</td><td>10</td><td>2</td><td>3</td></tr>
</table>

P.T.O

Q. No.	Questions	M	CO	BL
2.	A database assigns unique **record IDs** numbered from 1 to 5000. For system optimization, IDs that are divisible by 3, 5, or 7 are reserved for internal indexing and cannot be given to user records. How many valid record IDs can be assigned to users for internal indexing? **Solution:** **Solution** We use the **Inclusion–Exclusion Principle.** $N = 5000$ **Step 1: Count multiples individually** $\left\lfloor \frac{5000}{3} \right\rfloor = 1666, \quad \left\lfloor \frac{5000}{5} \right\rfloor = 1000, \quad \left\lfloor \frac{5000}{7} \right\rfloor = 714$ **Step 2: Subtract multiples of pairs (LCM)** $\left\lfloor \frac{5000}{15} \right\rfloor = 333, \quad \left\lfloor \frac{5000}{21} \right\rfloor = 238, \quad \left\lfloor \frac{5000}{35} \right\rfloor = 142$ **Step 3: Add back multiples of triple (LCM)** $\left\lfloor \frac{5000}{105} \right\rfloor = 47$ **Step 4: Apply Inclusion–Exclusion** $\#(\text{divisible by 3 or 5 or 7}) = (1666 + 1000 + 714) - (333 + 238 + 142) + 47$ $= 3380 - 713 + 47 = 2714$ **Step 5: Subtract from total IDs** $\#(\text{valid record IDs}) = 5000 - 2714 = 2286$	10	3	2

P.T.O

Q. No.	Questions	M	CO	BL
3.	(i). How many different 3 –digit numbers can be formed using the digits 1, 2, 3, 4, 5 without repetition? (ii). In how many ways can the letters of the word **MATH** be arranged? (iii). From 7 men, a committee of 3 is to be formed. In how many ways can this be done? (iv). How many different 4 – letter words can be formed using the letters of the word **BOOK**? (v). In how many ways can 5 different books be arranged on a shelf? **Solution:** (i) How many different 3–digit numbers can be formed using the digits 1, 2, 3, 4, 5 without repetition? Solution: First digit = 5 choices, second = 4, third = 3. Total = 5 × 4 × 3 = 60. (ii) In how many ways can the letters of the word MATH be arranged? Solution: All 4 letters are distinct. Total = 4! = 24. (iii) From 7 men, a committee of 3 is to be formed. In how many ways? Solution: Choosing without order → C(7,3) = 35. (iv) How many different 4–letter words can be formed using the letters of the word BOOK? Solution: Total permutations = 4!/2! = 12 (since two O's are identical). (v) In how many ways can 5 different books be arranged on a shelf? Solution: Total = 5! = 120.	10	3	1

P.T.O

Q. No.	Questions	M	CO	BL
4.	A city decides to plant trees along a highway. In the first year, 50 trees are planted. In the second year, 120 trees are planted. From the third year onwards, the number of trees planted in a year is equal to the sum of trees planted in the previous year and twice the number of trees planted two years earlier. 1. Formulate a recurrence relation for the number of trees planted in year n. 2. Find the number of trees planted in the n^{th} year. 3. Compute the total number of trees planted by the end of the 4th year. .	10	3	3

Solution:

Solution:

Let a_n denote the number of trees planted in the n-th year. Then the recurrence relation is

$$a_n = a_{n-1} + 2a_{n-2}, \quad n \geq 3$$

with initial conditions

$$a_1 = 50, \quad a_2 = 120.$$

Step 1: Solve the recurrence relation.
The characteristic equation is

$$r^2 - r - 2 = 0.$$

Factorizing,

$$(r-2)(r+1) = 0 \quad \Rightarrow \quad r = 2, -1.$$

Hence the general solution is

$$a_n = \alpha \cdot 2^{n-1} + \beta \cdot (-1)^{n-1}.$$

Step 2: Use initial conditions.
For $n = 1$:

$$a_1 = \alpha \cdot 2^0 + \beta \cdot (-1)^0 = \alpha + \beta = 50.$$

For $n = 2$:

$$a_2 = \alpha \cdot 2^1 + \beta \cdot (-1)^1 = 2\alpha - \beta = 120.$$

Step 3: Solve for α and β.
From $\alpha + \beta = 50$, we have $\beta = 50 - \alpha$.

Substitute into $2\alpha - \beta = 120$:

$$2\alpha - (50 - \alpha) = 120 \quad \Rightarrow \quad 3\alpha - 50 = 120.$$

So,

$$3\alpha = 170 \quad \Rightarrow \quad \alpha = \tfrac{170}{3}, \quad \beta = 50 - \tfrac{170}{3} = -\tfrac{20}{3}.$$

Step 4: Final formula.
Thus,

$$a_n = \frac{170}{3} \cdot 2^{n-1} - \frac{20}{3} \cdot (-1)^{n-1}.$$

Step 5: Compute specific values.
For $n = 3$:

$$a_3 = \frac{170}{3} \cdot 2^2 - \frac{20}{3} \cdot (-1)^2 = \frac{170}{3} \cdot 4 - \frac{20}{3} \cdot 1 = \frac{680}{3} - \frac{20}{3} = 220.$$

For $n = 4$:

$$a_4 = \frac{170}{3} \cdot 2^3 - \frac{20}{3} \cdot (-1)^3 = \frac{170}{3} \cdot 8 - \frac{20}{3} \cdot (-1) = \frac{1360}{3} + \frac{20}{3} = 460.$$

Step 6: Total number of trees in 4 years.

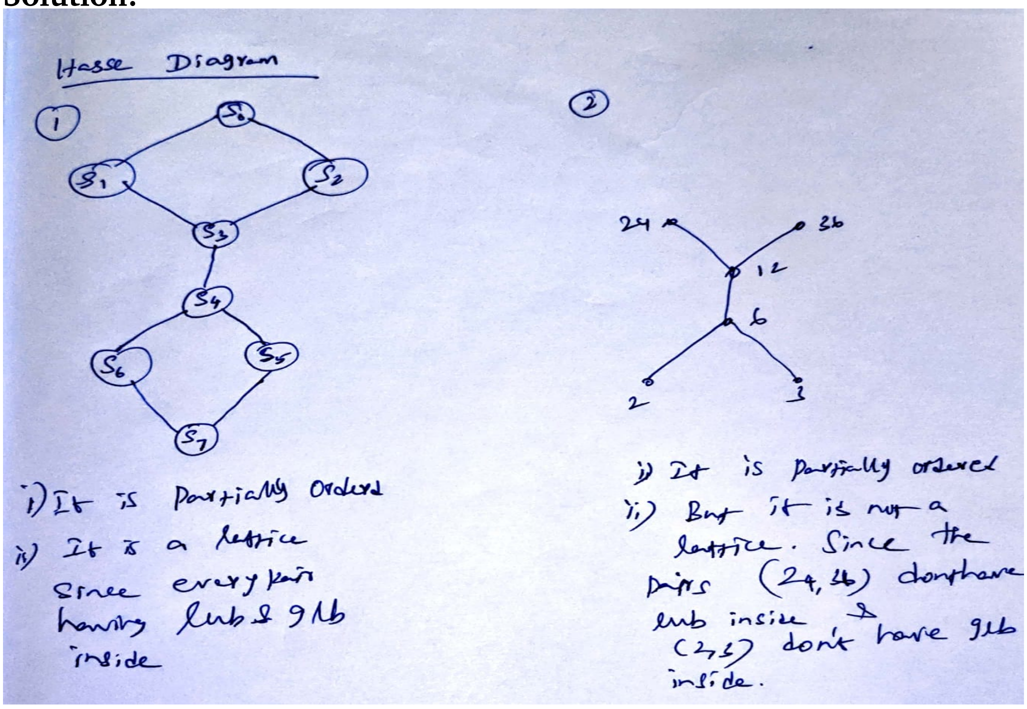
$$a_1 + a_2 + a_3 + a_4 = 50 + 120 + 220 + 460 = 850.$$

Final Answer:

$$a_3 = 220, \quad a_4 = 460, \quad \text{Total after 4 years} = 850.$$

Q. No.	Questions	M	CO	BL
5.	(1). Let $X_1 = \left\{S_0, S_1, S_2, \cdots, S_7\right\}$ where $S_0 = \{a,b,c,d,e,f\}$, $S_1 = \{a,b,c,d,e\}$, $S_2 = \{a,b,c,e,f\}$, $S_3 = \{a,b,c,e\}$, $S_4 = \{a,b,c\}$, $S_5 = \{a,b\}$, $S_6 = \{a,c\}$ and $S_7 = \{a\}$. (i). Verify whether $(X_1, \subseteq)$ is a partially ordered set or not. (ii). If so, draw a Hasse diagram for the same. (2). Consider the set $X_2 = \{2,3,6,12,24,36\}$ and let for $a, b \in X_2$, "$a \leq b$" be the relation "a divides b" *(i)*. Verify whether X_2 is a Poset or not. *(ii)*. Draw the Hasse diagram of $(X_2, \leq)$. *(iii)*. Check $(X_2, \leq)$ is lattice or not.	10	4	3

Solution:



****