



- KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE  
➤ DON'T WRITE ANYTHING ON THE QUESTION PAPER

Answer ALL Questions  
(10 X 10 = 100 Marks)

1. a) Let  $L1 = \{ n^2 \% 3 = 0 \text{ and } n \in \mathbb{Z} \}$ ,  $L2 = \{ n \% 3 = 0 \text{ and } n \in \mathbb{Z} \}$ . (Note: % is modulo operator). Prove by contrapositive, "If  $L1$  then  $L2$ ". [6]  
b) Let  $\Sigma = \{ \#, \$, \& \}$  be an alphabet. How many distinct strings of length 5 can be formed over  $\Sigma$ . [2]  
c) Let  $\Sigma = \{ a, b \}$  and consider the language [2]  
 $L1 = \{ w \in \Sigma^* \mid w \text{ begins with } a \text{ and } |w| \geq 3 \}$  and  
 $L1 - L2 = \{ aba, abb \}$   
Determine one possible definition for the language  $L2$  (using set builder notation or formal description) such that  $L1 - L2$  is satisfied.
2. a) Write a Regular Expression for a language that cannot have two consecutive a's together over  $\Sigma = \{ a, b \}$  [3]  
b) Based on the above regular expression which you obtained in the Question No. 2. (a), convert it into Non-Deterministic Finite Automata. [7]
3. Convert the following Context Free Grammar (CFG) into an equivalent Greibach Normal Form (GNF). Also prove the string "abab" is derivable in 4 steps of Right Most Derivation via the resultant GNF.  
$$S \rightarrow SS \mid aSb \mid ab$$
  
 $S$  is the start symbol of the CFG.
4. Design a Push Down Automata (PDA) for  
 $L = \{ a^i b^j c^k d^l \mid i=k \text{ or } j=l, i, j, k, l > 0 \}$  and justify whether your obtained PDA is deterministic (DPDA) or non-deterministic (NPDA)?

5. Design a Turing Machine (TM) that performs the following operation over  $\Sigma = \{0,1\}$ . Also briefly discuss about your idea for constructing transition rules and explain how you process example 3 or example 4.

Copy Paste operation

For Example 1 – Input : 0011

Output : 00110011

Example 2 – Input: 010010

Output: 010010010010

Example 3 – Input: 00

Output: 0000

Example 4 – Input: 111

Output : 111111

6. Either Prove or Disprove the following statements:

[3+4  
+3]

- i. If  $L$  and  $\bar{L}$  are Recursively Enumerable, then both languages are recursive
- ii. If  $L$  is recursive, then  $\bar{L}$  is also recursive.
- iii. If  $L$  and  $\bar{L}$  are Recursive, then both languages are recursively Enumerable.

7. a) A student X derives a string ( $w$ ) of length  $K$  from Chomsky Normal Form (CNF). An another student Y derives the same string ( $w$ ) of length  $K$  via Greibach Normal Form (GNF). Who would derive the string  $w$  quickly? X or Y? Justify. [3]

- b) Let  $L = \{w.w^R \mid w \in \{a,b\}^+\}$  is a context Free Language (CFL). (Note:  $w^R$  means reversal of  $w$ ). Answer the following with proper justification. [7]
- i. If you complement  $L$ , will  $\bar{L}$  be also a CFL?
  - ii. If you reverse  $L$ , will  $L^R$  be not a CFL?
  - iii. If you perform Kleene closure over  $L$ , will  $L^*$  be not a CFL?

8. a) Let us say a problem P1 is given to you. You need to find whether P1 is decidable or not. How do you find it? Suppose the given problem P1 is an instance of below-mentioned Post Correspondence Problem (PCP), what is your stand in it? [6]

List A: 1, 10111, 10 and List B : 111,10,0

- b) Draw Chomskian Hierarchy and interpret the relationship among grammars, languages and automata from the diagram. [4]

9.a)

Design a Turing Machine TM that computes

$$f(m,n) = \begin{cases} m - n & \text{if } m > n \\ \text{otherwise blank} & \end{cases}$$

Where  $m, n$  are integers over unary representation of  $\Sigma = \{0\}$ .

Also compute Instantaneous Description (ID) for  $f(0000,00)$  and  $f(00,000)$ .

OR

9.b)

Design a Turing Machine for unary to ternary (Base 3) convertor.

For Example 1 - Input: 00000

Output : 12

Example 2 - Input : 000

Output : 10

Example 3 - Input : 0000000000000000

Output: 120

10.a)

Let  $L1 = \{ w \mid \text{length of } w \text{ is not even} \}$  and  $L2 = \{ w \mid \text{length of } w \text{ is divisible by } 3 \}$  over  $\Sigma = \{0,1\}$ . Let  $M1$  and  $M2$  are the corresponding Deterministic Finite Automata (DFA) for  $L1$  and  $L2$ .

Find

- i)  $M1 \times M2$  (means cross product of  $M1$  and  $M2$ . That means DFA accepting  $L1 \cap L2$ ).
- ii) Complementation of  $M1 \times M2$ .
- iii) Reversal of  $M1 \times M2$ .

OR

10.b)

- i) Let  $M = (Q, \Sigma, \delta, q_0, F)$  be a Deterministic Finite Automata (DFA) [5]  
Statement 1 : If you reverse  $M$ , the result automata ( $M^R$ ) is either DFA or Non-Deterministic Finite Automata (NFA)

Statement 2: If you complement a DFA, the resultant language is also complemented.

Statement 3: If you complement a NFA, the resultant language need to be complemented always.

Analyse the truthness of Statement 1, 2 and 3 and justify your answer.

- ii) Let us assume the length of a string is  $n$ . If so, how many states are required for minimized DFA that accepts the following: [5]

- Exactly  $n$  length strings
- Atleast  $n$  length strings
- Atmost  $n$  length strings

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