REG.NO.:

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SCHOOL OF ADVANCED SCIENCES
CONTINUOUS ASSESSMENT TEST - I
WINTER SEMESTER 2024-2025

SLOT: B1+TB1

Programme Name & Branch : **B.Tech.**
Course Code and Course Name : **BMAT202L; Probability and Statistics**
Faculty Name(s) : **Common Slot QP**
Class Number(s) : **Common Slot QP**
Date of Examination : **28-Jan-2025 (Session 1)**
Exam Duration : **90 minutes** **Maximum Marks: 50**

General instruction(s):

- Answer All Questions
- M - Max mark; CO – Course Outcome; BL – Blooms Taxonomy Level (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create)
- Course Outcomes
  CO1: Compute and interpret descriptive statistics using numerical and graphical techniques
  CO2: Understand the basic concepts of random variables and find an appropriate distribution for analyzing data specific to an experiment
  CO3: Apply statistical methods like correlation

Q. No	Question	M	CO	BL
1.	In a factory employing 3000 persons, 5 percent earn less than Rs. 3 per hour, 580 persons earn Rs. 3.01 to Rs. 4.50 per hour, 30 percent earn from Rs. 4.51 to Rs. 6.00 per hour, 500 persons earn from Rs. 6.01 to Rs. 7.50 per hour, 20 percent earn from Rs. 7.51 to Rs. 9.00 per hour, and the rest earn Rs. 9.01 or more per hour. What are the mean, median, and mode wage?	10	1	3
2.	The following table shows the distribution of employees based on their weekly working hours: Weekly hours: 30-34 | 35-39 | 40-44 | 45-49 | 50-54 | 55-59 No. of employees: 4 | 6 | 10 | 14 | 8 | 8 For the given data, calculate quartile deviation, standard deviation and compare the results to determine which measure of dispersion is better.	10	1	2
3.	A random variable X has the following probability distribution: x: -2 | -1 | 0 | 1 | 2 | 3 $p(x)$: 0.1 | k | 0.2 | $2k$ | 0.3 | $3k$ (a) Find the value of k, (b) Evaluate $P(X<2)$ and $P(-2<X<2)$, (c) find the cdf of X and (d) evaluate the mean of X	10	2	3
4.	The joint pdf of a two-dimensional RV (X,Y) is given by $f(x,y) = xy^2 + \frac{x^2}{8},\ 0 \le x \le 2, 0 \le y \le 1$ Compute (i) $P(X > 1)$, (ii) $P(Y < 1/2)$, (iii) $P(X > 1/Y < 1/2)$, (iv) $P(X < Y)$ and (v) $P(X + Y \le 1)$.	10	2	4
5.	Find the coefficient of correlation between X and Y using the following data: X: 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 Y: 16 | 19 | 23 | 26 | 30 | 33 | 36 | 40 | 43	10	3	2

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Solution (Slot-B1)

Ans 1:

CALCULATIONS FOR MEDIAN WAGE

Earnings (in Rs.)	*Percentage of workers*	*No. of workers (f)*	*Less than c.f.*	*Class boundaries*
less than 3	5%	$\frac{5}{100} \times 3000 = 150$	150	Below 3·005
3·01—4·50	—	580	730	3·005—4·505
4·51—6·00	30 %	$\frac{30}{100} \times 3000 = 900$	1630	4·505—6·005
6·01—7·50	—	500	2130	6·005—7·505
7·51—9·00	20 %	$\frac{20}{100} \times 3000 = 600$	2730	7·505—9·005
9·01 and above		$3000 - 2730 = 270$	$3000 = N$	9·005 and above

$N/2 = 1500$. The *c.f.* just greater than 1500 is 1630. The corresponding class 4·51–6·00, whose class boundaries are 4·505–6·005, is the median class. Using the median formula, we get:

$$\text{Median} = l + \frac{h}{f}\left(\frac{N}{2} - C\right) = 4{\cdot}505 + \frac{1{\cdot}5}{900}(1500 - 730)$$
$$= 4{\cdot}505 + 1{\cdot}283 \approx 5{\cdot}79$$

Mean Wage = Rs. 6.06 per hour
Mode Wage = Rs. 5.17 per hour

Ans 2: $Q_1 = 40.75, Q_3 = 54.19, Q.D. = 6.72$
Standard Deviation = 7.35
Standard deviation is the better measure of dispersion in most cases, as it considers all values.

Ans 3: (a) k=1/15

(b) P(X<2) = P(X=-2)+ P(X=-1) + P(X=0)+ P(X=1)= ½
P(-2<X<2) = P(X=-1) + P(X=0)+ P(X=1)= 2/5

(c) F(x) = 0, when x < -2
= 1/10, when $-2 \leq x < -1$
=1/6, when $-1 \leq x < 0$
= 11/30, when $0 \leq x < 1$
=1/2, when $1 \leq x < 2$
= 4/5, when $2 \leq x < 3$
=1, when $3 \leq x$

(d) Mean = 16/15

Ans 4: (i)

$$P(X > 1) = \int\int_{\substack{R_1 \\ (x>1)}} f(x, y)\, dx\, dy$$

$$= \int_0^1 \int_1^2 \left(xy^2 + \frac{x^2}{8} \right) dx dy = \frac{19}{24}$$

(ii)

$$P(Y < 1/2) = \int\int_{\substack{R_2 \\ \left(y<\frac{1}{2}\right)}} \left(xy^2 + \frac{x^2}{8} \right) dx\, dy$$

$$= \int_0^{1/2} \int_0^2 \left(xy^2 + \frac{x^2}{8} \right) dx\, dy$$

$$= \frac{1}{4}$$

(iii)

$$P(X > 1, Y < 1/2) = \int\int_{\substack{R_3 \\ \left(x>1 \,\&\, y<\frac{1}{2}\right)}} \left(xy^2 + \frac{x^2}{8} \right) dx\, dy$$

$$= \int_0^{1/2} \int_1^2 \left(xy^2 + \frac{x^2}{8} \right) dx\, dy$$

$$= \frac{5}{24}$$

(iv)

$$P(X < Y) = \int\int_{\substack{R_4 \\ (x<y)}} \left(xy^2 + \frac{x^2}{8} \right) dx\, dy$$

$$= \int_0^1 \int_0^y \left(xy^2 + \frac{x^2}{8} \right) dx\, dy = \frac{53}{480}$$

(v)

$$P(X + Y \le 1) = \int\int_{\substack{R_5 \\ (x+y\le 1)}} \left(xy^2 + \frac{x^2}{8} \right) dx\, dy$$

$$= \int_0^1 \int_0^{1-y} \left(xy^2 + \frac{x^2}{8} \right) dx\, dy = \frac{13}{480}$$

Ans 5: r = 0.974