



VIT

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

REG.NO.:

**SCHOOL OF ADVANCED SCIENCES
CONTINUOUS ASSESSMENT TEST - I
WINTER SEMESTER 2024-2025**

SLOT: D1+TD1

Programme Name & Branch : B.Tech
Course Code and Course Name : Probability and Statistics- BMAT202L
Faculty Name(s) : Common Slot Question Paper
Class Number(s) : Common Slot Question Paper
Date of Examination : 30-01-2025
Exam Duration : 90 minutes **Maximum Marks: 50**

General instruction(s):

- Answer All Questions (5*10=50 Marks)

Q. No	Question	M	CO	BL																																								
1.	Calculate the mean, median and mode for the following distribution. <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td>Marks</td> <td>130-134</td> <td>135-139</td> <td>140-144</td> <td>145-149</td> <td>150-154</td> <td>155-159</td> <td>159-164</td> </tr> <tr> <td>No. of Students</td> <td>5</td> <td>15</td> <td>28</td> <td>24</td> <td>17</td> <td>10</td> <td>1</td> </tr> </table>	Marks	130-134	135-139	140-144	145-149	150-154	155-159	159-164	No. of Students	5	15	28	24	17	10	1	10	1	1																								
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No. of Students	5	15	28	24	17	10	1																																					
2.	Find the coefficient of mean deviation from mean, quartile deviation and coefficient of variation for the following data. <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td>X</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> </tr> <tr> <td>f</td> <td>4</td> <td>36</td> <td>100</td> <td>234</td> <td>280</td> <td>204</td> <td>112</td> <td>28</td> <td>4</td> </tr> </table>	X	0	1	2	3	4	5	6	7	8	f	4	36	100	234	280	204	112	28	4	10	1	1																				
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f	4	36	100	234	280	204	112	28	4																																			
3.	If the probability density function of a continuous random variable X is given by $f(x) = \begin{cases} ax, & 0 \leq x \leq 1, \\ a, & 1 \leq x \leq 2, \\ 3a - ax, & 2 \leq x \leq 3, \\ 0, & \text{elsewhere.} \end{cases}$ (i) Find the value of 'a'. (ii) Find the cumulative distribution function of X . (iii) Find $P(X \geq 1.5)$.	10	2	3																																								
4.	For the bivariate probability distribution of (X, Y) given below, find $P(X \leq 1)$, $P(Y \leq 3)$, $P(X \leq 1, Y \leq 3)$, $P(X \leq 1/Y \leq 3)$, $P(Y \leq 3/X \leq 1)$ and $P(X + Y \leq 4)$. <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="border: none;"></td> <td style="border: none;">Y</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td style="border: none;">X</td> <td style="border: none;"></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td style="border: none;">0</td> <td style="border: none;"></td> <td>0</td> <td>0</td> <td>1/32</td> <td>2/32</td> <td>2/32</td> <td>3/32</td> </tr> <tr> <td style="border: none;">1</td> <td style="border: none;"></td> <td>1/16</td> <td>1/16</td> <td>1/8</td> <td>1/8</td> <td>1/8</td> <td>1/8</td> </tr> <tr> <td style="border: none;">2</td> <td style="border: none;"></td> <td>1/32</td> <td>1/32</td> <td>1/64</td> <td>1/64</td> <td>0</td> <td>2/64</td> </tr> </table>		Y	1	2	3	4	5	6	X								0		0	0	1/32	2/32	2/32	3/32	1		1/16	1/16	1/8	1/8	1/8	1/8	2		1/32	1/32	1/64	1/64	0	2/64	10	2	3
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2		1/32	1/32	1/64	1/64	0	2/64																																					
5.	Compute the coefficient of correlation between X and Y using the following data: <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td>X</td> <td>65</td> <td>67</td> <td>66</td> <td>71</td> <td>67</td> <td>70</td> <td>68</td> <td>69</td> </tr> <tr> <td>Y</td> <td>67</td> <td>68</td> <td>68</td> <td>70</td> <td>64</td> <td>67</td> <td>72</td> <td>70</td> </tr> </table>	X	65	67	66	71	67	70	68	69	Y	67	68	68	70	64	67	72	70	10	3	2																						
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BMAT 202L

CAT-1 KEY

DI+TDI

(5)

(1)

Class	f	cf	Mid
129.5 - 134.5	5	5	132.5
134.5 - 139.5	15	20	137.5
139.5 - 144.5	28	48	142.5
<u>144.5 - 149.5</u>	<u>24</u>	<u>72</u>	147.5
149.5 - 154.5	17	89	152.5
154.5 - 159.5	10	99	157.5
159.5 - 164.5	1	100	162.5

$$\sum f = N = 100$$

$$\frac{N}{2} = 50$$

(3)

$$\text{Mean} = \frac{\sum xf}{\sum f} = \frac{132.5 \times 5 + 137.5 \times 15 + 142.5 \times 28 + \dots + 162.5 \times 1}{100}$$

(3)

$$\text{Mean} = 145.85 \approx 146 \quad (\text{OR}) = A + \frac{\sum fd}{N} h$$

$$= 145.85 \approx 146$$

$$\text{Median} = L + \frac{h}{f} \left(\frac{N}{2} - c \right)$$

$$= 144.5 + \frac{5}{24} (50 - 48) = 144.92 \approx 145$$

(2)

$$\text{Mode} = L + \frac{h(f_0 - f_1)}{2f_0 - f_1 - f_2}$$

$$= 139.5 + \frac{5(28 - 15)}{2(28) - 15 - 24}$$

$$= 143.32 \approx 143$$

(2)

(2)

— x — x — x —

$$\bar{x} = \frac{\sum fx}{N}, \quad N = \sum f$$

$$\text{Mean Deviation} = \frac{\sum f |D|}{N}$$

$$\text{co-efficient of Mean Deviation} = \frac{MD}{\text{Median}}$$

x	f	fx	$ x - \bar{x} $	$f x - \bar{x} = D $	$f D ^2$
0	4	0	3.97	15.88	63.044
1	36	36	2.97	106.92	317.552
2	100	200	1.97	197	388.09
3	234	702	0.97	226.98	220.171
4	280	1120	0.03	8.4	0.252
5	204	1020	1.03	210.12	216.424
6	112	672	2.03	227.36	461.541
7	28	196	3.03	84.84	257.065
8	4	32	4.03	16.12	64.967
$N = 1002$		$\sum fx = 3978$			

$$\bar{x} = \frac{\sum fx}{N} = \frac{3978}{1002} = 3.97$$

$$MD = \frac{\sum f|D|}{N} = \frac{1093.62}{1002} = 1.091$$

$$(1) \text{ Co-eff of MD} = \frac{1.091}{3.97} = 0.275$$

$$\sum f|D| = 1093.62$$

$$\sum f|D|^2 = 1989.106$$

$$\sigma^2 = \frac{\sum f|D|^2}{N} = \frac{1989.106}{1002} = 1.985, \sigma = 1.409$$

(2)

(ii) Quartile Deviation

$$Q = \frac{Q_3 - Q_1}{2}$$

$$Q_1 = 3$$

$$\frac{N}{2} = 501$$

$$\frac{3N}{4} = 751.5$$

$$Q_3 = 5$$

$$\therefore Q = \frac{5 - 3}{2} = 1$$

(2)

(iii) Co-efficient of variation

$$= 100 \times \frac{\sigma}{\bar{x}} = \frac{100 \times 1.409}{3.97} = 35.491$$

(3)

— x — x — x —

$$(3) (i) \int_0^1 a n \, dn + \int_1^2 a \, dn + \int_2^3 (3a - a n) \, dn = 1$$

(3)

(2)

$$a \left(\frac{1}{2} + 1 + \frac{1}{2} \right) = 1 \Rightarrow \boxed{a = \frac{1}{2}}$$

$$(ii) -\infty < n \leq 0, \quad F(n) = 0$$

$$0 \leq n < 1, \quad F(n) = \frac{n^2}{4}$$

$$1 \leq n < 2, \quad F(n) = \frac{n}{2} - \frac{1}{4}$$

$$2 \leq n < 3, \quad F(n) = \frac{3}{2}n - \frac{n^2}{4} - \frac{5}{4}$$

$$3 \leq n < \infty, \quad F(n) = 1$$

(6)

$$(iii) \int_{1.5}^2 f(n) \, dn = P(X \geq 1.5) = \int_{1.5}^3 f(n) \, dn$$

$$= \int_{1.5}^2 f(n) \, dn + \int_2^3 f(n) \, dn = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} //$$

(2)

(4)

	Y	1	2	3	4	5	6	$P(X=x_i)$
X	0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$	$\frac{8}{32}$
	1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{10}{16}$
	2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$	$\frac{8}{64}$
($Y=0$)		$\frac{3}{32}$	$\frac{3}{32}$	$\frac{11}{64}$	$\frac{13}{64}$	$\frac{6}{32}$	$\frac{14}{64}$	1

$$(i) P(X \leq 1) = P(X=0) + P(X=1) = \frac{7}{8}$$

(1)

$$(ii) P(Y \leq 3) = P(Y=1) + P(Y=2) + P(Y=3) = \frac{23}{64}$$

(2)

(4) 5

(iii) $P(X \leq 1, Y \leq 3) = (0 + 0 + \frac{1}{32}) + (\frac{1}{16} + \frac{1}{16} + \frac{1}{8}) = \frac{9}{32}$ (2)

(iv) $P(X \leq 1 | Y \leq 3) = \frac{9/32}{23/64} = \frac{18}{23}$ (2)

(v) $P(Y \leq 3 | X \leq 1) = \frac{9/32}{7/8} = \frac{9}{28}$ (1)

(vi) $P(X + Y \leq 4) = \frac{3}{32} + \frac{1}{4} + \frac{1}{16} = \frac{13}{32}$ (2)

(5)

Correlation coefficient

$\gamma_{xy} = 0.3974$

(10)

— x — x — x —