U/D/TY

Reg. No:

Final Assessment Test – April 2025

VIT® Vellore Institute of Technology (Deemed to be University under section 3 of the UGC Act, 1956)

Course: **BECE207L - Random Processes**

Class NBR(s): **0631 / 0642 / 0646 / 0654 / 4700**

Slot: **A1+TA1**

Time: **Three Hours**

Max. Marks: **100**

- **KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTIC**
- **DON'T WRITE ANYTHING ON THE QUESTION PAPER**

Answer ALL Questions

(10 X 10 = 100 Marks)

1. The joint density function of two random variables X and Y is given by

$$f_{XY}(x,y) = \begin{cases} 6e^{-(2x+3y)}; 0 \le x \le \infty \ and \ 0 \le y \le \infty \\ 0; otherwise \end{cases}$$

Find the marginal densities $f_X(x)$ and $f_Y(y)$ and comment whether X and Y are independent.

2. Two random variables X and Y have the following joint probability density function:

$$f_{XY}(x,y) = \begin{cases} xe^{-x(y+1)} & 0 \le x \le \infty; 0 \le y \le \infty \\ 0 \text{ ; otherwise} \end{cases}$$

Find the conditional density function $f_{xy}\left(\frac{x}{y}\right)$ and $f_{xy}\left(\frac{y}{x}\right)$.

3. The zero mean Gaussian random variables X_1 and X_2 having covariance matrix $[C_{XX}] = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ are transformed to new variables $Y_1 = 5X_1 + 2X_2$ and $Y_2 = -X_1 + 3X_2$. Find the mean, variance, covariance and correlation coefficient of the random variables Y_1 and Y_2.

4. The joint probability mass function of two random variables X and Y is given by

$$P_{XY}(x,y) = \begin{cases} \frac{1}{42}(2x+y); x = 0,1,2 \ and \ y = 0,1,2,3 \\ 0; otherwise \end{cases}$$

Find the correlation coefficient between X and Y.

5. The random process $X(t) = U + V\cos(\omega t + \theta)$, where U and V are independent random variables with mean 0 and 3, variance 3 and 4 respectively. The random variable θ is independent and uniformly distributed over $\left[\frac{-\pi}{2}, \frac{3\pi}{2}\right]$ and ω is a constant. Find the mean and autocorrelation of $X(t)$. Is $X(t)$ a WSS process?

6. If $X(t)$ and $Y(t)$ are stationary and statistically independent random process with autocorrelation functions $R_{XX}(\tau) = 2e^{-2|\tau|}\cos(\omega\tau)$ and $R_{YY}(\tau) = 9 + e^{-3|\tau|}$. Let $Z(t) = AX(t)Y(t)$, where A is independent random variable with mean and variance 2 and 9, respectively. Find mean, variance and autocorrelation of $Z(t)$.

7. The power spectral density of $X(t)$ is given by

$$S_{XX}(\omega) = \begin{cases} 1 + \omega^2, & |\omega| \leq 1 \\ 0, & |\omega| > 1 \end{cases}$$

Find the autocorrelation of the process.

8. If $X(t)$ is a stationary random process with zero mean and autocorrelation $R_{XX}(\tau) = e^{-2|\tau|}$ is applied to system with the transfer function $H(\omega) = \frac{1}{2+j\omega}$.

Find the mean and power spectral density of the output process.

9.(a) An amplifier has three stages, the effective noise temperature of each stage is 150K, 350K and 600K respectively. Available power gain of first stage is 10 and overall input effective noise temperature is 190K.

i. What is the available power gain of the second stage?

ii. Find the standard spot noise figure.

iii. Find the operating spot noise figure when used with a source of noise temperature of 50K?

OR

9.(b) An additive white Gaussian noise X(t) is applied to a network shown in the Figure 1. Find the output power spectral density of the network.

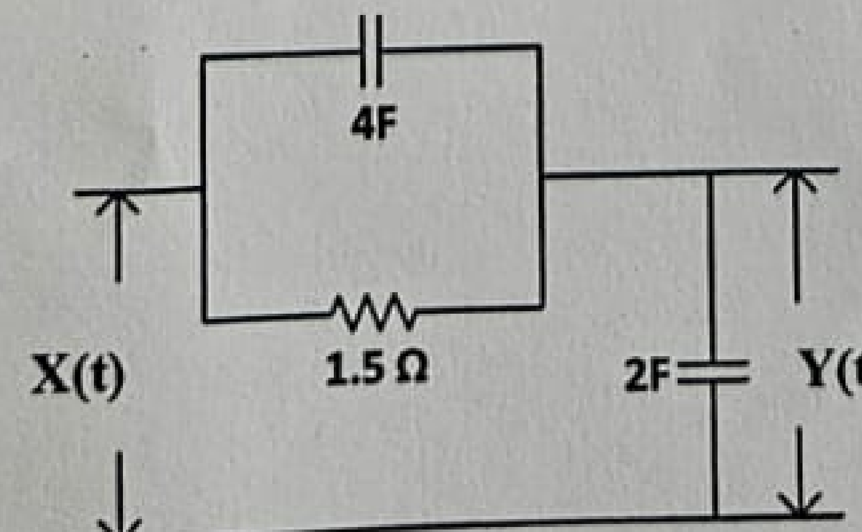

Figure 1

10.(a) The power spectral density of $X(t)$ is given by

$$S_{XX}(\omega) = \begin{cases} \frac{1}{2}\left(1 + \cos\left(\frac{\pi\omega}{W}\right)\right) & ; |\omega| \le W \\ 0 & ; |\omega| > W \end{cases}$$

Draw the power spectrum as function of ω. Compute the noise bandwidth and compare that with its 3-dB bandwidth.

OR

10.(b) A signal $X(t)$ is received at the front end of the receiver as $X(t) = s(t) + w(t)$, where $s(t)$ is a pulse of amplitude A with time duration T and $w(t)$ is a white Gaussian noise with power spectrum $\frac{N_0}{2}$. Design a matched filter to extract $s(t)$ such that signal to noise ratio is maximized. If the output of matched filter is observed at time $t = T$, what will be $\left(\frac{S}{N}\right)_o$?

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