REG.NO.:

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

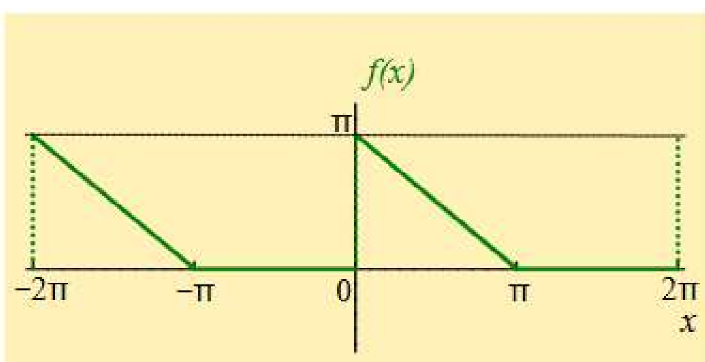
SCHOOL OF ELECTRONICS ENGINEERING
CONTINUOUS ASSESSMENT TEST - II
FALL SEMESTER 2025-2026

SLOT: B1+TB1

Programme Name & Branch	**:B.Tech -ECE and BML**
Course Code and Course Name	**:BECE202L Signals and Systems**
Faculty Name(s)	**: Dr. KARTHIKEYAN, Dr. RAJESH N, Dr. NAWAZ SHAFI, Dr. BIJAYLAXMI DAS, Dr. ASHISH P, Dr. JAGANA BIHARI PADHY**
Class Number(s)	**: VL2025260100448,2647,2648,2650,3478,3489**
Date of Examination	**: 06-10-2025**
Exam Duration	**: 90 minutes** **Maximum Marks: 50**

General instruction(s):

- Answer All Questions
- M - Max mark; CO – Course Outcome; BL – Blooms Taxonomy Level (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create)
- Course Outcomes
  1. Differentiate between various types of signals and understand the implication of operations on signals.
  2. Understand terms like causal, dynamic, linear, time invariant and stability of systems; also compute impulse response of both continuous-time and discrete-time systems.

Q. No	Question	M	CO	BL
1.	Sketch and compute the trigonometric Fourier series co-efficient for the following signal. $f(x) = \begin{cases} 0 & \pi < x < 2\pi \\ \pi - x & 0 < x < \pi \end{cases}$ and has the period of 2π. 	10	3	2

$$
\begin{aligned}
a_0 &= \frac{1}{\pi}\int_0^{2\pi} f(x)\,\mathrm{d}x \\
&= \frac{1}{\pi}\int_0^{\pi} (\pi - x)\,\mathrm{d}x + \frac{1}{\pi}\int_{\pi}^{2\pi} 0\cdot \mathrm{d}x \\
&= \frac{1}{\pi}\left[\pi x - \frac{1}{2}x^2\right]_0^{\pi} + 0 \\
&= \frac{1}{\pi}\left[\pi^2 - \frac{\pi^2}{2} - 0\right] \\
\text{i.e. } a_0 &= \frac{\pi}{2}.
\end{aligned}
$$

$$
\begin{aligned}
a_n &= \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,\mathrm{d}x \\
&= \frac{1}{\pi}\int_0^{\pi} (\pi - x)\cos nx\,\mathrm{d}x + \frac{1}{\pi}\int_{\pi}^{2\pi} 0\cdot \mathrm{d}x \\
\text{i.e. } a_n &= \underbrace{\frac{1}{\pi}\left\{\left[(\pi - x)\frac{\sin nx}{n}\right]_0^{\pi} - \int_0^{\pi}(-1)\cdot\frac{\sin nx}{n}\,\mathrm{d}x\right\}}_{\text{using integration by parts}} + 0 \\
&= \frac{1}{\pi}\left\{(0-0) + \int_0^{\pi}\frac{\sin nx}{n}\mathrm{d}x\right\} \quad \text{, see TRIG} \\
&= \frac{1}{\pi n}\left[\frac{-\cos nx}{n}\right]_0^{\pi} \\
&= -\frac{1}{\pi n^2}\left(\cos n\pi - \cos 0\right) \\
\text{i.e. } a_n &= -\frac{1}{\pi n^2}\left((-1)^n - 1\right) \quad \text{, see TRIG}
\end{aligned}
$$

	$$\text{i.e. } a_n = \begin{cases} 0 & , n \text{ even} \\ \frac{2}{\pi n^2} & , n \text{ odd} \end{cases}$$ STEP THREE $$\begin{aligned} b_n &= \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,\mathrm{d}x \\ &= \frac{1}{\pi}\int_0^{\pi}(\pi - x)\sin nx\,\mathrm{d}x + \int_{\pi}^{2\pi} 0\cdot\mathrm{d}x \\ &= \frac{1}{\pi}\left\{\left[(\pi - x)\left(-\frac{\cos nx}{n}\right)\right]_0^{\pi} - \int_0^{\pi}(-1)\cdot\left(-\frac{\cos nx}{n}\right)\mathrm{d}x\right\} + 0 \\ &= \frac{1}{\pi}\left\{\left(0 - \left(-\frac{\pi}{n}\right)\right) - \frac{1}{n}\cdot 0\right\}, \text{ see TRIG} \\ \text{i.e. } b_n &= \frac{1}{n}. \end{aligned}$$			
2.	Let $x(t) = \sum_{n=-\infty}^{+\infty} \delta\left[t - n\frac{T}{2}\right]$. Compute the Fourier series coefficient of $x(t)$ using exponential Fourier series and compute the Fourier series coefficients of $y(t) = \frac{d^2x(t)}{dt^2}$ using exponential Fourier series properties.	10	3	2

B1 Slot

1. Let $x(t) = \sum_{n=-\infty}^{+\infty} \delta\left(t - n\frac{T}{2}\right)$. Compute the Fourier series coefficients of $x(t)$ and also $y(t) = \frac{d^2x(t)}{dt^2}$ using exponential Fourier series property.

Solution: $x(t) = \sum_{n=-\infty}^{+\infty} \delta\left(t - n\frac{T}{2}\right)$; $T_0 = \frac{T}{2}$, $\omega_0 = \frac{2\pi}{T_0} = \frac{4\pi}{T}$

F.S. coeff of $x(t)$ is $a_k = \frac{2}{T}\int_{-T/4}^{+T/4} \delta(t) e^{-jk\frac{4\pi}{T}t}\, dt$

$a_k = \frac{2}{T} \ \forall k$

Using exponential F.S., $x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{j\left(\frac{4\pi}{T}\right)kt}$

$$\frac{d^2x(t)}{dt^2} = \sum_{k=-\infty}^{+\infty} \underbrace{-\left(\frac{4\pi}{T}\right)^2 k^2 a_k}_{d_k} e^{j\frac{4\pi}{T}kt}$$

d_k is the F.S. coeff. of $\frac{d^2x(t)}{dt^2}$ (i.e. $y(t)$).

$$d_k = -\left(\frac{4\pi}{T}\right)^2 k^2 a_k$$
$$= -\left(\frac{4\pi}{T}\right)^2 k^2 \frac{2}{T} \Bigg|_{a_k = \frac{2}{T}}$$

Let $x[n] = \sum_{k=-\infty}^{+\infty} \delta[n - 5k]$. Compute the Fourier series coefficients of $x[n]$ using exponential Fourier series and also compute $\frac{1}{N}\sum_{n=<N>} |x[n]|^2$ using exponential Fourier series properties.

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B2 slot

1. let $x[n] = \sum_{k=-\infty}^{+\infty} \delta[n-5k]$. Compute the Fourier series coefficients of $x[n]$ and also compute $\frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2$ using exponential Fourier series property.

Solution:

$$x[n] = \sum_{k=-\infty}^{+\infty} \delta[n-5k];$$

$$N = 5, \quad \omega_0 = \frac{2\pi}{N} = \left.\frac{2\pi}{5}\right|_{N=5}$$

F.S. coeff. of $x[n]$ is $a_k = \frac{1}{5} \; \forall k$.

In F.S, parseval's relation:

$$\frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2 = \sum_{k=\langle N\rangle} |a_k|^2$$

$$= \sum_{k=0}^{4} |a_k|^2 \quad \Big| \text{ given } N=5, \; k=0,1,2,3,4$$

$$= |a_0|^2 + |a_1|^2 + |a_2|^2 + |a_3|^2 + |a_4|^2$$

$$= \left|\frac{1}{5}\right|^2 + \left|\frac{1}{5}\right|^2 + \left|\frac{1}{5}\right|^2 + \left|\frac{1}{5}\right|^2 + \left|\frac{1}{5}\right|^2$$

Given $a_k \; \forall N$,

$$\frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2 = \frac{1}{5}$$

3.	Find the Fourier transform of the following signals: i. $x(t) = e^{-\lvert t\rvert} \sin 5\lvert t\rvert \quad for\ all\ t$ ii. $x(t) = e^{-a\lvert t\rvert} sgn(t)$ 1.i)	10	4	3

Given $$x(t) = e^{-|t|} \sin 5|t| \text{ for all } t$$

i.e. $$x(t) = \begin{cases} e^{-(-t)} \sin 5(-t) = -e^{t} \sin 5t & \text{for } t<0 \\ e^{-(t)} \sin 5(t) = e^{-t} \sin 5t & \text{for } t>0 \end{cases}$$

i.e. $$x(t) = -e^{t} \sin 5t\, u(-t) + e^{-t} \sin 5t\, u(t)$$

$$\therefore \quad X(\omega) = \int_{-\infty}^{\infty} \left[-e^{t} \frac{(e^{j5t} - e^{-j5t})}{2j} u(-t) + e^{-t} \left(\frac{e^{j5t} - e^{-j5t}}{2j} \right) u(t) \right] e^{-j\omega t}\, dt$$

$$= -\frac{1}{2j} \left\{ \int_{-\infty}^{0} \left[e^{[1-j(\omega-5)]t} - e^{[1-j(\omega+5)]t} \right] dt \right\}$$

$$+ \frac{1}{2j} \left\{ \int_{0}^{\infty} \left[e^{-[1+j(\omega-5)]t} - e^{-[1+j(\omega+5)]t} \right] dt \right\}$$

Fourier Transforms | 331

$$= -\frac{1}{2j} \left\{ \int_{0}^{\infty} \left[e^{-[1-j(\omega-5)]t} - e^{-[1-j(\omega+5)]t} \right] dt \right\}$$

$$+ \frac{1}{2j} \left\{ \int_{0}^{\infty} \left[e^{-[1+j(\omega-5)]t} - e^{-[1+j(\omega+5)]t} \right] dt \right\}$$

$$= -\frac{1}{2j} \left[\frac{e^{-[1-j(\omega-5)]t}}{-[1-j(\omega-5)]} - \frac{e^{-[1-j(\omega+5)]t}}{-[1-j(\omega+5)]} \right]_0^{\infty}$$

$$+ \frac{1}{2j} \left[\frac{e^{-[1+j(\omega-5)]t}}{-[1+j(\omega-5)]} - \frac{e^{-[1+j(\omega+5)]t}}{-[1+j(\omega+5)]} \right]_0^{\infty}$$

$$= -\frac{1}{2j} \left[\frac{1}{1-j(\omega-5)} - \frac{1}{1-j(\omega+5)} \right] + \frac{1}{2j} \left[\frac{1}{1+j(\omega-5)} - \frac{1}{1+j(\omega+5)} \right]$$

$$= -\frac{1}{2j} \left[\frac{1-j\omega-j5-1+j\omega-j5}{(1-j\omega)^2+5^2} \right] + \frac{1}{2j} \left[\frac{1+j\omega+j5-1-j\omega+j5}{(1+j\omega)^2+5^2} \right]$$

$$= \frac{5}{(1-j\omega)^2+5^2} + \frac{5}{(1+j\omega)^2+5^2} \quad \text{[neglecting impulses]}$$

ii)

$$x(t) = e^{-a|t|} \operatorname{sgn}(t)$$

$$x(t) = e^{-a|t|} \operatorname{sgn}(t) = \begin{cases} e^{-a(-t)}(-1) = -e^{at} & \text{for } t<0 \\ e^{-a(t)}(1) = e^{-at} & \text{for } t>0 \end{cases}$$

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	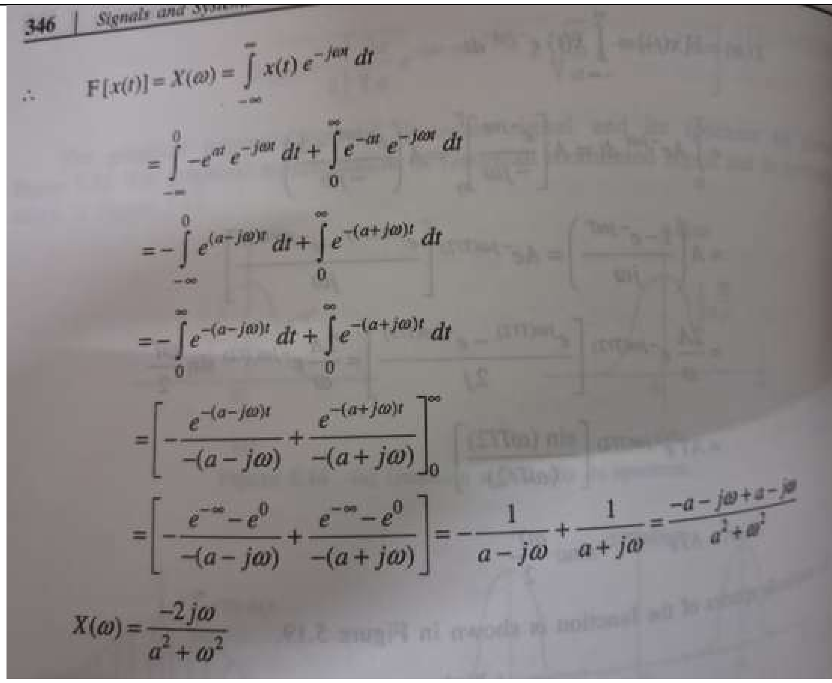 346 | Signals and Sys... $\therefore\quad F[x(t)] = X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$ $= \int_{-\infty}^{0} -e^{at} e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt$ $= -\int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$ $= -\int_{0}^{\infty} e^{-(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$ $= \left[-\frac{e^{-(a-j\omega)t}}{-(a-j\omega)} + \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^{\infty}$ $= \left[-\frac{e^{-\infty}-e^{0}}{-(a-j\omega)} + \frac{e^{-\infty}-e^{0}}{-(a+j\omega)} \right] = -\frac{1}{a-j\omega} + \frac{1}{a+j\omega} = \frac{-a-j\omega+a-j\omega}{a^2+\omega^2}$ $X(\omega) = \frac{-2j\omega}{a^2+\omega^2}$			
4.	**Let the signal y(t) be defined as:** $\boldsymbol{y(n) = x(n-4)e^{j\frac{\pi}{2}n} + x(n).sin2\omega_0 n}$ **Where:** $\boldsymbol{x(n) = a^n u(n), 0 < a < 1}$ **Find the Fourier Transform $Y(e^{j\omega})$ of y(t) without directly evaluating any integrals. Clearly state and justify each property used.** Ans: 1. DTFT of $x[n] = a^n u[n]$: $X(e^{j\omega}) = 1 / (1 - a e^{-j\omega})$, for $\|a\| < 1$ 2. First term: $x[n - 4] \cdot e^{j\pi/2\, n}$ - Time-shifting by 4 → Multiply DTFT by $e^{-j\omega \cdot 4}$ - Modulation by $e^{j\pi/2\, n}$ → Frequency shift by $\pi/2$ ⇒ DTFT = $X(e^{j(\omega - \pi/2)}) \cdot e^{-j4\omega}$ 3. Second term: $x[n] \cdot \sin(2\omega_o n)$ - $\sin(2\omega_o n) = 0.5j(e^{j2\omega_o n} - e^{-j2\omega_o n})$ → Frequency shift by $\pm 2\omega_o$ ⇒ DTFT = $0.5j\, [X(e^{j(\omega - 2\omega_o)}) - X(e^{j(\omega + 2\omega_o)})]$ 4. Final DTFT: $Y(e^{j\omega}) = X(e^{j(\omega - \pi/2)}) \cdot e^{-j4\omega} + 0.5j\, [X(e^{j(\omega - 2\omega_o)}) - X(e^{j(\omega + 2\omega_o)})]$	10	4	2

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5.	i. If the operation $F[.]$ denotes the Fourier Transform, evaluate the following (using suitable properties wherever necessary) $F\left[\frac{1}{2\pi(1+t^2)}\right]$ ii. The input and impulse response of a discrete-time LTI system are given by $x[n] = \delta[n] + \delta[n-2]$ and $h[n] = \left(\frac{1}{3}\right)^n u[n]$, respectively. Obtain $Y(e^{jw})$ and the time domain output signal $y[n]$ using frequency domain techniques.	10	4	3

we know that,

$$\frac{e^{-a|t|}}{2a} \xleftrightarrow{F.T.} \frac{1}{a^2+\omega^2}$$

As per duality,

If $x(t) \longleftrightarrow X(\omega)$

$X(t) \longleftrightarrow 2\pi\, x(-\omega)$

$$\therefore \frac{1}{a^2+t^2} \xleftrightarrow{F} 2\pi \frac{e^{-a|\omega|}}{2a}$$

$$\therefore F\left[\frac{1}{a^2+t^2}\right] = \frac{2\pi\, e^{-a|\omega|}}{2a}$$

put $a=1$ $\quad F\left[\frac{1}{1+t^2}\right] = \pi e^{-|\omega|}$

Divide by 2π on b.s.

$$F\left[\frac{1}{2\pi(1+t^2)}\right] = \frac{\pi e^{-|\omega|}}{2\pi} = \frac{1}{2} e^{-|\omega|}$$

$$\therefore F\left[\frac{1}{2\pi(1+t^2)}\right] = \frac{e^{-|\omega|}}{2}$$

iii.

4

$x[n] = \delta(n) + \delta(n-2)$ $\quad H(e^{j\omega}) = \frac{1}{1-\frac{1}{3}e^{-j\omega}}$

$X(e^{j\omega}) = 1 + e^{-j\omega 2}$

$$\therefore Y(e^{j\omega}) = X(e^{j\omega})\, H(e^{j\omega}) = (1+e^{-j\omega 2}) \cdot \frac{1}{1-\frac{1}{3}e^{-j\omega}} = \ldots$$

$$= \left(\frac{1}{3}\right)^n u(n) + \left(\frac{1}{3}\right)^{n-2} \cdot u(n-2)$$

Time shifting

S. No	Emp. ID	Faculty Name	Sign
1	16395	KARTHIKEYAN M	
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3	19716	JAGANA BIHARI PADHY	

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