REG.NO.:

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF ELECTRONICS ENGINEERING
CONTINUOUS ASSESSMENT TEST - 2
WINTER SEMESTER 2024-2025

SLOT: B1+TB1

Programme Name & Branch	:	**B.Tech - ECE**
Course Code and Course Name	:	**(BECE317L) Wireless and Mobile Communications**
Faculty Name(s)	:	**Budhaditya Bhattacharyya, Abhijit Bhowmick, Rajkishor Kumar, Hemanta Kumar Sahu, Tangudu Ramji, Vijaya Durga Chintala**
Class Number(s)	:	**VL2024250500671/674/676/ 678/679/ 680**
Date of Examination	:	**17/01/2025**
Exam Duration	:	**90 minutes** **Maximum Marks: 50**

General instruction(s):

- Answer All Questions
- M - Max mark; CO – Course Outcome; BL – Blooms Taxonomy Level (3 – Apply, 4 – Analyse)
- **CO-3** Demonstrate the principles of multicarrier modulation.
- **CO-4** Select a suitable diversity technique to combat the multipath fading effects.
- **CO-5** Identify suitable MIMO techniques to enhance the spectrum efficiency.

Q. No	Question	M	CO	BL
1.	A Wi-Fi system using a 20 MHz bandwidth employs a 1024-subcarrier OFDM system. Compute: i. The bandwidth of each sub-carrier i. The OFDM symbol duration without a cyclic prefix ii. The OFDM symbol duration with a 16.67% cyclic prefix iii. The number of samples in the cyclic prefix iv. The percentage loss in spectral efficiency due to CP • Wi-Fi System Bandwidth = 20 MHz • Number of Sub-carriers (N) = 1024 • Cyclic Prefix Length = 16.67% (0.1667 of symbol duration) **(a) Bandwidth of Each Sub-carrier** $B_{\text{sub}} = \frac{\text{Total Bandwidth}}{\text{Number of Sub-carriers}} = \frac{20\text{ MHz}}{1024}$ $B_{\text{sub}} = 19.53125\text{ kHz}$ **(b) OFDM Symbol Time Without Cyclic Prefix** $T_s = \frac{1}{B_{\text{sub}}} = \frac{1}{19.53125 \times 10^3}$ $T_s = 51.2\ \mu\text{s}$	10	3	3

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	(c) OFDM Symbol Time With Cyclic Prefix (16.67%) Cyclic prefix duration: $$T_{CP} = 0.1667 \times T_s = 0.1667 \times 51.2$$ $$T_{CP} = 8.53\ \mu s$$ Total symbol duration with CP: $$T_{total} = T_s + T_{CP} = 51.2 + 8.53 = 59.73\ \mu s$$ **(d) Number of Samples in the Cyclic Prefix** Since there are 1024 sub-carriers, the number of samples in the cyclic prefix is: $$N_{CP} = 0.1667 \times 1024 = 170.67 \approx 171$$ **(e) Loss in Spectral Efficiency Due to CP** $$\text{Loss} = \frac{T_{CP}}{T_{total}} \times 100\% = \frac{8.53}{59.73} \times 100 = 14.29\%$$			
2.	Consider an OFDM system with 512 sub-carriers and 24 wireless channel tap with SNR of 25 dB. Compute the BER of the OFDM system. Calculate the percentage change in estimated BER, when channel SNR drops to 20 dB under the effect of frequency offset. Will there be any change in the calculated SINR if the system suffers 6 % carrier frequency offset? Comment on your finding. **Step 1: Convert SNR from dB to Linear Scale** The SNR in linear scale is given by: $$\gamma = 10^{\frac{\text{SNR (dB)}}{10}}$$ $$\gamma = 10^{\frac{25}{10}} = 10^{2.5} = 316.23$$ **Step 2: BER Calculation in Rayleigh Fading** For Rayleigh fading, the average BER for different modulations is given by: **BER for BPSK in Rayleigh Fading** $$BER_{\text{BPSK}} = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma}{1+\gamma}}\right)$$ Substituting $\gamma = 316.23$: $$BER_{\text{BPSK}} = \frac{1}{2}\left(1 - \sqrt{\frac{316.23}{1+316.23}}\right)$$ $$= \frac{1}{2} \times \left(1 - \sqrt{\frac{316.23}{317.23}}\right)$$ $$= \frac{1}{2} \times (1 - 0.9984)$$ $$= 0.000789$$	10	3	3
3.	Consider an L=2 receive antenna system with BPSK modulation having instantaneous channel coefficients that are $h_1 = 2 - j$ and $h_2 = 1 + 2j$ The received signals at different antennas are $y_1(k)$, and $y_2(k)$corrupted by noise $n_1(k)$and $n_2(k)$ which are Gaussian-	[2.5*4] = 10	4	3

distributed random variables with variance $\sigma_n^2 = 1$. Estimate the following:

i. Consider an Equal Gain Combining (EGC) diversity technique applied at the receiver and derive the optimal weight vector.

(3) $L=2$, $h_1 = 2-j$, $h_2 = 1+2j$, $\sigma_n^2 = 1$

(i) $|h_1| = \sqrt{5}$, $|h_2| = \sqrt{5}$

$$W_{opt} = \begin{bmatrix} h_1/|h_1| \\ h_2/|h_2| \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{5}} - \frac{j}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} + \frac{2j}{\sqrt{5}} \end{bmatrix}$$

ii. If the SNR at the output of the EGC combiner is found to be 10 dB, then find $E\{|x(k)|^2\}$.

(ii) SNR = 10 dB = 10 (linear)

$$SNR_{EGC} = \left(\sum_{i=1}^{2} |h_i|\right)^2 \times \frac{E\{|x(k)|^2\}}{L \times \sigma_n^2}$$

$$\Rightarrow 10 = \left(\sqrt{5}+\sqrt{5}\right)^2 \times \frac{E\{|x(k)|^2\}}{2 \times 1}$$

$$\Rightarrow E\{|x(k)|^2\} = 1.$$

iii. Calculate the instantaneous BER at the output of the EGC combiner.

(iii) Inst. BER $= Q\left(\sqrt{SNR_{EGC}}\right)$

$= Q\left(\sqrt{10}\right) = 0.008$

$\approx \frac{1}{2} e^{-10/2} = 3.3 \times 10^{-3}$

iv. If a third antenna is added in the EGC combiner with channel coefficient $h_3 = 1 + j$, evaluate the effects of the instantaneous BER compared to the previous configuration. Calculate the percentage improvement in BER performance.

	$h_3 = 1+j$, $\lvert h_3 \rvert = \sqrt{2}$ $\left(\sum_{i=1}^{3} \lvert h_i \rvert\right)^2 = (\sqrt{5}+\sqrt{5}+\sqrt{2})^2 = 34.647$ $SNR = \frac{34.647 \times 1}{3 \times 1} = 11.549$. [illegible] $BER = Q(\sqrt{11.549}) = 1.55 \times 10^{-3}$ Improvement $= \left(\frac{3.36 \times 10^{-3} - 1.55 \times 10^{-3}}{3.36 \times 10^{-3}}\right) \times 100$ $= 53.90\%$.			
4.	a) Find the difference in SNR (dB) for a wireless system with L= 4 for MRC and wireline channel to achieve a BER $=10^{-8}$. Wireline $BER = Q(\sqrt{SNR}) = \frac{1}{2} e^{-\frac{SNR}{2}}$ $\Rightarrow 10^{-8} = \frac{1}{2} e^{-\frac{SNR}{2}}$ $\Rightarrow SNR = 35.455$ $= 15.496$ dB. For $L=4$ $BER_{MRC} = {}^{2L-1}C_L \left(\frac{1}{2SNR}\right)^L$ $= {}^{7}C_4 \left(\frac{1}{2SNR}\right)^4$ $\Rightarrow 10^{-8} = \frac{35}{16 (SNR)^4}$ $\Rightarrow SNR_{MRC} = 121.617 = 20.849$ dB $\boxed{\text{Difference} = 5.353 \text{ dB}}$ b) For a SIMO communication system, find the number of receiving antennas if the probability of deep fade is 1.3×10^{-3} with SNR = 13 dB.	[5+5] = 10	4	3

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	(b) $SNR = 13\,dB = 20$ (linear) $P_{H} = 1.3\times10^{-3}$ $\Rightarrow \frac{1}{L!}\left(\frac{1}{SNR}\right)^{L} = 0.0013$ $\Rightarrow \frac{1}{L!}\left(\frac{1}{20}\right)^{L} = 0.0013$ $L=1 \quad \Rightarrow \frac{1}{1}\times\frac{1}{20} = 0.05 \neq 0.0013$ $L=2 \quad \Rightarrow \frac{1}{2!}\left(\frac{1}{20}\right)^{2} = 0.00125 \simeq 0.0013$ $L=3 \quad \frac{1}{3!}\left(\frac{1}{20}\right)^{3} = 2.85\times10^{-5} \neq 0.0013$ $\boxed{L=2 \text{ is required}}$			
5	Consider a MIMO channel matrix H is given as $$H = \begin{bmatrix} 1 & -1 \\ 2 & 2 \\ -1 & 3 \end{bmatrix}$$ If two symbols, x_1 and x_2, are transmitted simultaneously, the receive signal and AWGN noise at each antenna are denoted by y_i and w_i, respectively, where i = 1, 2 and 3. Noises are independent and identically distributed with a variance of -3 dB. Find the following for zero-forcing receiver: i) Find the estimated signals $\widehat{x_1}$ and $\widehat{x_2}$ at the receiver. ii) Find the noise covariance. $\Rightarrow$ i) $H = \begin{bmatrix} 1 & -1 \\ 2 & 2 \\ -1 & 3 \end{bmatrix}$ $\quad H^T = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 2 & 3 \end{bmatrix}$ $H^TH = \begin{bmatrix} 6 & 0 \\ 0 & 14 \end{bmatrix}$ $\quad \therefore (H^TH)^{-1} = \begin{bmatrix} 1/6 & 0 \\ 0 & 1/14 \end{bmatrix}$ $(H^TH)^{-1}H^T = \begin{bmatrix} 1/6 & 0 \\ 0 & 1/14 \end{bmatrix}\begin{bmatrix} 1 & 2 & -1 \\ -1 & 2 & 3 \end{bmatrix}$ $\quad \hat{x} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}$ $= \begin{bmatrix} 1/6 & 2/6 & -1/6 \\ -1/14 & 2/14 & 3/14 \end{bmatrix}$ $\quad \bar{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ $\therefore \hat{x} = (H^TH)^{-1}H^T\bar{y}$ $\Rightarrow \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \begin{bmatrix} 1/6 & 2/6 & -1/6 \\ -1/14 & 2/14 & 3/14 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ $\therefore$ Estimated signals are $\hat{x}_1 = \frac{1}{6}\bar{y}_1 + \frac{2}{6}\bar{y}_2 - \frac{1}{6}\bar{y}_3$ $\hat{x}_2 = -\frac{1}{14}\bar{y}_1 + \frac{2}{14}\bar{y}_2 + \frac{3}{14}\bar{y}_3$ ii) $\sigma_w^2 = -3\,dB = 10^{-3/10} = 1/2$ $R_{ww} = E[ww^H] = \begin{bmatrix} \sigma_w^2 & 0 & 0 \\ 0 & \sigma_w^2 & 0 \\ 0 & 0 & \sigma_w^2 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$	[5+5] = 10	5	3

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