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SCHOOL OF ADVANCED SCIENCES
CONTINUOUS ASSESSMENT TEST - II
WINTER SEMESTER 2025-2026

SLOT: A2+TA2+TAA2

Programme Name & Branch	**: B.Tech.**
Course Code and Course Name	**: IAMAT202 - Discrete Mathematics and Graph Theory**
Faculty Name(s)	**: Dr. A. Chandrasekar, Dr. Arun Choudhary, Dr. R. Palanivel**
Class Number(s)	**: Common to A2 slot**
Date of Examination	**: 15/03/2026**
Exam Duration	**: 90 minutes** **Maximum Marks: 50**

General instruction(s):

- Answer All Questions
- M - Max mark; CO – Course Outcome; BL – Blooms Taxonomy Level (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create)
- Course Outcomes (Type the CO statements covered in this question paper. Use the CO number as per the syllabus copy)
  CO-2 Relate algebraic structures to enhance problem-solving techniques
  CO-3 Solve engineering problems involving counting principles.
  CO-4 Understand the fundamentals of graph theory, including trees and connectivity

<table>
<tr><th>Q. No</th><th>Question</th><th>M</th><th>CO</th><th>BL</th></tr>
<tr><td>1.</td><td>Find the canonical CNF and canonical DNF of the Boolean function
$$f(x, y, z) = (x + y)(y + z)(x' + z').$$</td><td>10</td><td>2</td><td>2</td></tr>
<tr><td>2.</td><td>In a university of 300 students, 180 students like Mathematics, 150 students like Physics, 130 students like Chemistry, 90 students like both Mathematics and Physics, 70 students like both Physics and Chemistry, 60 students like both Mathematics and Chemistry, 40 students like all three subjects. Then find
(a) How many students like at least one subject?
(b) How many students like none of the subjects?
(c) How many students like exactly one subject?
(d) How many students like exactly two subjects?</td><td>10</td><td>3</td><td>3</td></tr>
<tr><td>3.</td><td>Use the method of generating function to solve the recurrence relation
$$a_n = 2a_{n-1} - 5, \; n \geq 1$$
with initial condition $a_0 = 3$.</td><td>10</td><td>3</td><td>3</td></tr>
<tr><td rowspan="2">4.</td><td>a) At a recent math seminar, 9 mathematicians greeted each other by shaking hands. Is it possible that each mathematician shook hands with exactly 7 people at the seminar?</td><td>5</td><td rowspan="2">4</td><td>2</td></tr>
<tr><td>b) Determine if the following two graphs are isomorphic:
Graph $G_1: V = \{1, 2, 3, 4\}, \; E = \{(1,2), (2,3), (3,4), (4,1)\}$.
Graph $G_2: V = \{a, b, c, d\}, \; E = \{(a, c), (c, b), (b, d), (d, a)\}$.
Show your reasoning based on vertex degrees and adjacency.</td><td>5</td><td>2</td></tr>
<tr><td rowspan="2">5.</td><td>a) Consider a graph G representing a map of five cities where every city is connected to every other city (Complete Graph K_5).
I. Does this graph contain an Euler Circuit? Justify.
II. Determine if the graph is Hamiltonian and find one Hamiltonian Cycle.</td><td>6</td><td rowspan="2">4</td><td>3</td></tr>
<tr><td>b) In a graph with 6 vertices, what is the maximum number of edges the graph can have if it is disconnected?.</td><td>4</td><td>2</td></tr>
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Solution 1: The canonical DNF and canonical CNF of the function

$$f(x, y, z) = (x + y)(y + z)(x' + z').$$

Canonical DNF
$f = x'yz' + x'yz + xyz'$
Canonical CNF
$f = (x + y + z)(x + y + z')(x' + y + z)(x' + y + z')(x' + y' + z')$
Solution : 2 In a university of 300 students, the following data is collected:

- 180 students like Mathematics (M)
- 150 students like Physics (P)
- 130 students like Chemistry (C)
- 90 students like both Mathematics and Physics
- 70 students like both Physics and Chemistry
- 60 students like both Mathematics and Chemistry
- 40 students like all three subjects

(a) How many students like at least one subject?

Using the Inclusion–Exclusion Principle:

$$\begin{aligned} |M \cup P \cup C| &= |M| + |P| + |C| - |M \cap P| - |P \cap C| - |M \cap C| + |M \cap P \cap C| \\ &= 180 + 150 + 130 - 90 - 70 - 60 + 40 \\ &= 280 \end{aligned}$$

(b) How many students like none of the subjects?
None $= 300 - 280 = 20$
(c) How many students like exactly one subject?
Only Mathematics: $180 - 90 - 60 + 40 = 70$
Only Physics:$150 - 90 - 70 + 40 = 30$
Only Chemistry: $130 - 70 - 60 + 40 = 40$
Total exactly one: $70 + 30 + 40 = 140$
(d) How many students like exactly two subjects?
Mathematics and Physics only: $90 - 40 = 50$
Physics and Chemistry only: $70 - 40 = 30$
Mathematics and Chemistry only: $60 - 40 = 20$
Total exactly two: $50 + 30 + 20 = 100$
Solution 3: Define the Generating Function
Let

$$G(x) = \sum_{n=0}^{\infty} a_n x^n.$$

$$\sum_{n=1}^{\infty} a_n x^n = 2\sum_{n=1}^{\infty} a_{n-1} x^n - 5\sum_{n=1}^{\infty} x^n.$$

G(x)= $\dfrac{3-8x}{(1-x)(1-2x)}$

G(x)= $\dfrac{5}{1-x} - \dfrac{2}{1-2x}$

$$G(x) = \sum_{n=0}^{\infty} (5 - 2\cdot 2^n)\, x^n.$$

$$\boxed{a_n = 5 - 2^{n+1}.}$$

Solution 4(a): It seems like this should be possible. Each mathematician chooses one person to not shake hands with. But this cannot happen. We are asking whether a graph with 9 vertices can have each vertex have degree 7. If such a graph existed, the sum of the degrees of the vertices would be $9 \cdot 7 = 63$. This would be twice the number of edges (handshakes) resulting in a graph with 31:5 edges. That is impossible. Thus, at least one (in fact, an odd number) of the mathematicians must have shaken hands with an even number of people at the seminar.

Solution 4(b): 1. Degree Sequence: Both graphs have 4 vertices, and every vertex in both graphs has a degree of 2.

2. Structural Mapping: We can define a bijection $f : V(G_1) \to V(G_2)$ such that $f(1) = a, f(2) = c, f(3) = b, f(4) = d$.

3. Adjacency Check: In G_1, 1 is adjacent to 2 and 4. In G_2, $f(1) = a$ is adjacent to $f(2) = c$ and $f(4) = d$. Adjacency is preserved.

Conclusion: The graphs are isomorphic.

Solution 5(a): 1. Euler Circuit: A connected graph has an Euler circuit if and only if every vertex has an even degree. In K_5, each of the 5 vertices has a degree of $n - 1 = 4$. Since all degrees are even, the graph contains an Euler circuit.

2. Hamiltonian Cycle: A graph is Hamiltonian if there is a cycle that visits every vertex exactly once. For K_5 with vertices $\{A, B, C, D, E\}$, a valid Hamiltonian cycle is: $A \to B \to C \to D \to E \to A$.

Solution 5(b): In a graph with 6 vertices, what is the maximum number of edges the graph can have if it is disconnected?.Solution: 1. To maximize edges while remaining disconnected, the graph should consist of a complete component of $n - 1$ vertices and one isolated vertex.2. For $n = 6$, we create a complete graph K_5 using 5 vertices and leave the 6th vertex isolated.3. Edges in $K_5 = \frac{5(5-1)}{2} = \frac{20}{2} = \mathbf{10}$ edges.