QP & Key

VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF ADVANCED SCIENCES

CONTINUOUS ASSESSMENT TEST - I

FALL SEMESTER 2025-2026

SLOT: A2+TA2+TAA2

Course Code and Course Name : BAMAT205 - Discrete Mathematics and Linear Algebra

Date of Examination : 27-January -2026

Exam Duration : 90 minutes **Maximum Marks: 50**

Q. No	Question	M

1\. Determine whether the statement $[(p \to q) \wedge \sim q] \to \sim p$ is a tautology or not, using logical equivalences. (4)

Step	Expression	Logical Law Used
1	$[(p \to q) \wedge \neg q] \to \neg p$	Given statement
2	$[(\neg p \vee q) \wedge \neg q] \to \neg p$	Implication: $p \to q \equiv \neg p \vee q$
3	$[(\neg p \wedge \neg q) \vee (q \wedge \neg q)] \to \neg p$	Distributive law
4	$(\neg p \wedge \neg q) \to \neg p$	Contradiction: $q \wedge \neg q \equiv \bot$
5	$\neg(\neg p \wedge \neg q) \vee \neg p$	Implication: $A \to B \equiv \neg A \vee B$
6	$(p \vee q) \vee \neg p$	De Morgan's law
7	$(p \vee \neg p) \vee q$	Commutative law
8	$\top \vee q$	Law of excluded middle
9	$\top$	Domination law

Since the statement simplifies to $\top$ (true under all truth assignments), the given proposition is a tautology.

Obtain the principal disjunctive normal form of the following statement (6)

$$(P \to (Q \wedge R)) \wedge (\sim P \to (\sim Q \wedge \sim R))$$

Method 1: Using Logical Equivalences

Step	Expression	Law Used
1	$(P \to (Q \wedge R)) \wedge (\neg P \to (\neg Q \wedge \neg R))$	Given
2	$(\neg P \vee (Q \wedge R)) \wedge (P \vee (\neg Q \wedge \neg R))$	Implication law
3	$[\neg P \wedge (P \vee (\neg Q \wedge \neg R))] \vee [(Q \wedge R) \wedge (P \vee (\neg Q \wedge \neg R))]$	Distributive law
4	$(\neg P \wedge P) \vee (\neg P \wedge \neg Q \wedge \neg R) \vee (P \wedge Q \wedge R) \vee (Q \wedge R \wedge \neg Q \wedge \neg R)$	Distributive law
5	$(\neg P \wedge \neg Q \wedge \neg R) \vee (P \wedge Q \wedge R)$	Contradiction law
6	$(P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R)$	Commutative law

PDNF (by logical equivalences): $(P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R)$

Method 2: Using Truth Table

P	Q	R	$Q \land R$	$\neg Q \land \neg R$	$P \to (Q \land R)$	$\neg P \to (\neg Q \land \neg R)$	Result
T	T	T	T	F	T	T	T
T	T	F	F	F	F	T	F
T	F	T	F	F	F	T	F
T	F	F	F	T	F	T	F
F	T	T	T	F	T	F	F
F	T	F	F	F	T	F	F
F	F	T	F	F	T	F	F
F	F	F	F	T	T	T	T

Rows where the statement is true give the minterms: P ∧ Q ∧ R, ¬P ∧ ¬Q ∧ ¬R

2. Prove or disprove the validity of the following arguments:

"It is not sunny this afternoon and it is colder than yesterday"; "We will go to the playground only if it is sunny". "If we do not go to the ground, then we will go to a movie" and "If we go to a movie, then we will return home by sunset" lead to the conclusion *"We will return home by sunset".*

Let:

- S: It is sunny this afternoon
- C: It is colder than yesterday
- P: We will go to the playground
- M: We will go to a movie
- H: We will return home by sunset

The given premises can be written as:

1. $\neg S \land C$
2. $P \to S$
3. $\neg P \to M$
4. $M \to H$

The conclusion to be proved is:

H

Using Rules of Inference

Step	Statement	Reason
1	$\neg S \land C$	Premise
2	$\neg S$	Simplification (from 1)
3	$P \to S$	Premise
4	$\neg P$	Modus Tollens (2, 3)
5	$\neg P \to M$	Premise
6	M	Modus Ponens (4, 5)
7	$M \to H$	Premise
8	H	Modus Ponens (6, 7)

Since the conclusion H logically follows from the given premises using valid rules of inference, the argument is valid.

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3	Construct an argument using rules of inference to show that the hypotheses *"One student in this university knows how to use AI in study"* and *"Everyone who knows how to use AI in their study can get a high-paying job"* imply the conclusion *"Someone in this university can get a high-paying job"*. Let the domain be the set of all students in this university. Define the predicates: • $A(x)$: x knows how to use AI in study • $H(x)$: x can get a high-paying job The hypotheses are: 1. $\exists x\, A(x)$ 2. $\forall x\,(A(x) \to H(x))$ The conclusion to be proved is: $\exists x\, H(x)$ Using rules of inference	10

Step	Statement	Reason
1	$\exists x\, A(x)$	Premise
2	$\forall x\,(A(x) \to H(x))$	Premise
3	$A(c)$	Existential Instantiation (from 1)
4	$A(c) \to H(c)$	Universal Instantiation (from 2)
5	$H(c)$	Modus Ponens (from 3, 4)
6	$\exists x\, H(x)$	Existential Generalization (from 5)

Since the conclusion ∃x H(x) logically follows from the given hypotheses using valid rules of inference, the argument is valid.

4. By the principle of inclusion-exclusion, find the number of integers between 1 and 250 both inclusive that are not divisible by any of the integers 2, 3, 5 and 7. **(10)**

Let A, B, C, D be the sets of integers that lie between 1 and 250 and that are divisible by 2, 3, 5, and 7 respectively.

The elements of A are 2, 4, 6, …, 250

$\therefore$ $|A| = 125$, which is the same as $\left\lfloor \frac{250}{2} \right\rfloor$

Similarly, $|B| = \left\lfloor \frac{250}{3} \right\rfloor = 83$; $|C| = \left\lfloor \frac{250}{5} \right\rfloor = 50$, $|D| = \left\lfloor \frac{250}{7} \right\rfloor = 35$.

The set of integers between 1 and 250 which are divisible by 2 and 3, viz., $A \cap B$ is the same as that which is divisible by 6, since 2 and 3 are relatively prime numbers.

	$\therefore \quad |A \cap B| = \left\lfloor \frac{250}{6} \right\rfloor = 41$ Similarly, $|A \cap C| = \left\lfloor \frac{250}{10} \right\rfloor = 25$; $|A \cap D| = \left\lfloor \frac{250}{14} \right\rfloor = 17$ $|B \cap C| = \left\lfloor \frac{250}{15} \right\rfloor = 16$; $|B \cap D| = \left\lfloor \frac{250}{21} \right\rfloor = 11$; $|C \cap D| = \left\lfloor \frac{250}{35} \right\rfloor = 7$; $|A \cap B \cap C| = \left\lfloor \frac{250}{30} \right\rfloor = 8$; $|A \cap B \cap D| = \left\lfloor \frac{250}{42} \right\rfloor = 5$; $|A \cap C \cap D| = \left\lfloor \frac{250}{70} \right\rfloor = 3$; $|B \cap C \cap D| = \left\lfloor \frac{250}{105} \right\rfloor = 2$; $|A \cap B \cap C \cap D| = \left\lfloor \frac{250}{210} \right\rfloor = 1$ By the Principle of Inclusion-Exclusion, the number of integers between 1 and 250 that are divisible by at least one of 2, 3, 5 and 7 is given by $|A \cup B \cup C \cup D| = \{|A| + |B| + |C| + |D|\} - \{|A \cap B| + \cdots + |C \cap D|\} + \{|A \cap B \cap C| + \cdots + |B \cap C \cap D|\} - \{|A \cap B \cap C \cap D|\}$ $= (125 + 83 + 50 + 35) - (41 + 25 + 17 + 16 + 11 + 7) + (8 + 5 + 3 + 2) - 1$ $= 293 - 117 + 18 - 1 = 193$ $\therefore$ Number of integers between 1 and 250 that are not divisible by any of the integers 2, 3, 5 and 7 $=$ Total no. of integers $- |A \cup B \cup C \cup D|$ $= 250 - 193 = 57.$	
5.	Use the method of generating function to solve the recurrence relation $a_{n+1} - 8a_n + 16a_{n-1} = 4^n$; $n \geq 1$; $a_0 = 1, a_1 = 8$. Let the generating functions of $\{a_n\}$ be $G(x) = \sum_{n=0}^{\infty} a_n x^n$ Multiplying both sides of the given R.R. by x^n and summing up, we have $\sum_{n=1}^{\infty} a_{n+1} x^n - 8 \sum_{n=1}^{\infty} a_n x^n + 16 \sum_{n=1}^{\infty} a_{n-1} x^n = \sum_{n=1}^{\infty} (4x)^n$ i.e., $\frac{1}{x}\{G(x) - a_0 - a_1 x\} - 8\{G(x) - a_0\} + 16x\, G(x) = \frac{1}{1-4x} - 1$ i.e., $(1 - 8x + 16x^2)\, G(x) - a_0 - a_1 x + 8a_0 x = \frac{4x^2}{1-4x}$	10

i.e., $G(x) = \dfrac{a_0 + (a_1 - 8a_0)x}{(1-4x)^2} + \dfrac{4x^2}{(1-4x)^3}$

$= \dfrac{1}{(1-4x)^2} + \dfrac{4x^2}{(1-4x)^3}$, on using the values of a_0 and a_1.

$= (1 - 4x + 4x^2)\,(1 - 4x)^{-3}$

i.e., $\sum_{n=0}^{\infty} a_n x^n = (1 - 4x + 4x^2) \cdot \dfrac{1}{2}\{1 \cdot 2 + 2 \cdot 3\,(4x) + 3 \cdot 4(4x)^2 + \cdots + (n+1)\,(n+2)\,(4x)^n \cdots\}$

$\therefore$ $a_n = \dfrac{1}{2}[(n+1)\,(n+2)\,4^n - n(n+1)4^n + (n-1)n\,4^{n-1}]$

$= \dfrac{1}{2}4^{n-1}\{4(n^2 + 3n + 2) - 4(n^2 + n) + (n^2 - n)\}$

$= \dfrac{1}{2}(n^2 + 7n + 8) \cdot 4^{n-1}$.

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