REG.NO.:

VIT Vellore Institute of Technology (Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF ADVANCED SCIENCES
CONTINUOUS ASSESSMENT TEST - II
FALL SEMESTER 2025-2026

SLOT: A1+TA1+TAA1

Programme Name & Branch	**: B.Tech.**
Course Code and Course Name	**: BAMAT205 & Discrete Mathematics and Linear Algebra**
Faculty Name(s)	**: Common**
Class Number(s)	**: Common**
Date of Examination	**: 27.01.2026**
Exam Duration	**: 90 minutes** **Maximum Marks: 50**

1. Without constructing the truth tables, find the principal disjunctive normal form (PDNF) and principal conjunctive normal form (PCNF) of the statement

$$(p \vee \neg(q \vee r)) \vee (((p \wedge q) \wedge \neg r) \wedge p)$$

Solution:

Let $S \equiv (p \vee \neg(q \vee r)) \vee ((p \wedge q) \wedge \neg r) \wedge p$

$\equiv (p \vee (\neg q \vee \neg r)) \vee (p \wedge q \wedge \neg r \wedge p)$

$\equiv p \wedge (q \vee \neg q) \vee (\neg q \wedge \neg r) \vee (p \wedge q \wedge \neg r)$

$\equiv (p \wedge q) \vee (p \wedge \neg q) \vee (\neg q \wedge \neg r) \vee (p \wedge q \wedge \neg r)$

$\equiv ((p \wedge q) \wedge (r \vee \neg r)) \vee ((p \wedge \neg q) \wedge (r \vee \neg r)) \vee ((\neg q \wedge \neg r) \wedge (p \vee \neg p)) \vee (p \wedge q \wedge \neg r)$

$\equiv (p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r) \vee (p \wedge q \wedge \neg r)$

$\equiv (p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r) \quad (1)$

In (1), we have got the PDNF of S.

Now $\neg S \equiv (\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$

$\therefore \quad S \equiv \neg\neg S \equiv (p \vee \neg q \vee \neg r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q \vee \neg r) \quad (2)$

(2) is the required PCNF of S.

2. (a) Symbolize the following statement with and without the set of all positive integers as its domain.
"Given any positive integers, there is a greater positive integer."

Solution:

With positive integers as domain: ∀x ∃y (y > x).
Without positive integers as domain: ∀x (P(x) → ∃y (P(y) ∧ y > x)).

2. (b) Show that the set of premises $(p \to q) \wedge (r \to s), (q \to t) \wedge (s \to u), \neg(t \wedge u)$ and $(p \to r)$ implies $\neg p$.

Solution:

Step No.	*Statement*	*Reason*
1.	$(p \to q) \wedge (r \to s)$	P
2.	$p \to q$	T, 1 and simplification
3.	$r \to s$	T, 1 and simplification
4.	$(q \to t) \wedge (s \to u)$	P
5.	$q \to t$	T, 4 and simplification
6.	$s \to u$	T, 4 and simplification
7.	$p \to t$	T, 2, 5 and hypothetical syllogism
8.	$r \to u$	T, 3, 6 and hypothetical syllogism
9.	$p \to r$	P
10.	$p \to u$	T, 8, 9 and hypothetical syllogism
11.	$\urcorner t \to \urcorner p$	T and 7
12.	$\urcorner u \to \urcorner p$	T and 10
13.	$(\urcorner t \vee \urcorner u) \to \urcorner p$	T, 11, 12, and $(a \to b), (c \to b) \Rightarrow (a \vee c) \to b$
14.	$\urcorner(t \wedge u) \to \urcorner p$	T, 13 and De Morgan's law
15.	$\urcorner(t \wedge u)$	P
16.	$\urcorner p$	T, 14, 15 and modus ponens.

3. Show that the premises, “Every student who submits the assignment passes the course”, “Every student in this class submits the assignment or drops the course”, “No student in this class drops the course” and “There exists a student in this class” imply the conclusion, “There exists a student in this class who passes the course”.

Solution:

Let the domain be the set of all students in the institution.

$S(x)$: x is a student in this class

$A(x)$: x submits the assignment

$D(x)$: x drops the course

$P(x)$: x passes the course

Premises:

H_1 : $\forall x \left(A(x) \longrightarrow P(x)\right)$

H_2 : $\forall x \left(S(x) \longrightarrow (A(x) \vee D(x))\right)$

H_3 : $\forall x \left(S(x) \longrightarrow \neg D(x)\right)$

H_4 : $\exists x\, S(x)$

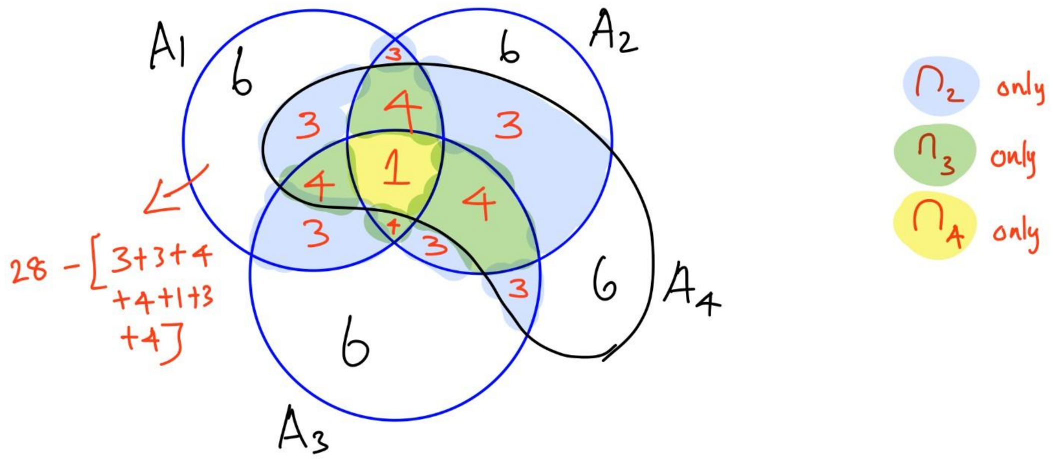
Conclusion:

C : $\exists x \left(S(x) \wedge P(x)\right)$

Step	Statement	Reason
1	$\forall x \left(S(x) \longrightarrow A(x) \vee D(x) \right)$	Rule P
2	$S(a) \longrightarrow (A(a) \vee D(a))$	Rule US
3	$\exists x\, S(x)$	Rule P
4	$S(a)$	Rule ES
5	$A(a) \vee D(a)$	2 & 4, Modus ponens
6	$\forall x \left(S(x) \longrightarrow \neg D(x)\right)$	Rule P
7	$S(a) \longrightarrow \neg D(a)$	Rule US
8	$\neg D(a)$	4 & 7, Modus Ponens
9	$A(a)$	5 & 8, Disjunctive Syllogism
10	$\forall x \left(A(x) \longrightarrow P(x)\right)$	Rule P

Step	Statement	Reason
11	$A(a) \longrightarrow P(a)$	Rule US
12	$P(a)$	9 & 11, Modus Ponens
13	$S(a) \wedge P(a)$	4 & 12, Conjunction
14	$\exists x \left(S(x) \wedge P(x)\right)$	Rule EG

4. **Solution:**



(a) The no. of elements belong to none of the four subsets

$= |U| - |A_1 \cup A_2 \cup A_3 \cup A_4|$

$= |U| - \left[\sum |A_i| - \sum |A_i \cap A_j| + \sum |A_i \cap A_j \cap A_k| - |A_1 \cap A_2 \cap A_3 \cap A_4| \right]$

$= 75 - [4(28) - 6(12) + 4(5) - 1]$

$= \boxed{16}$

(b) The no. of elements belong A_1 or A_2 or A_3 but not A_4 $= 6+3+6+3+4+3+6$

$= \boxed{31}$

(c) The no. of elements belong to exactly three of the four subsets $= 4+4+4+4$

$= \boxed{16}$

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5. **Solution:**

Let the generating function of $\{a_n\}$ be $G(x) = \sum_{n=0}^{\infty} a_n x^n$.

Multiplying both sides of the given R.R. by x^n and summing up, we have

$$\sum_{n=2}^{\infty} a_n x^n = 4\sum_{n=2}^{\infty} a_{n-1} x^n - 4\sum_{n=2}^{\infty} a_{n-2} x^n + \sum_{n=2}^{\infty} 4^n x^n$$

i.e, $\{G(x) - a_0 - a_1 x\} = 4x\{G(x) - a_0\} - 4x^2\, G(x) + \dfrac{1}{1-4x} - 1 - 4x.$

i.e., $(1 - 4x + 4x^2)\, G(x) = \dfrac{1}{1-4x} - 1 - 4x + 2 \quad (\because a_0 = 2 \text{ and } a_1 = 8)$

$\therefore$ $G(x) = \dfrac{1+(1-4x)^2}{(1-2x)^2 \cdot (1-4x)}$

$= \dfrac{4}{1-4x} - \dfrac{2}{(1-2x)^2}$, on splitting into partial fractions

i.e., $G(x) = \sum_{n=0}^{\infty} a_n x^n = 4[1 + 4x + (4x)^2 + \ldots + (4x)^n + \cdots \infty]$

$- 2[1 + 2 \cdot (2x) + 3 \cdot (2x)^2 + \cdots + (n+1)(2x)^n + \cdots \infty]$

$\therefore$ $a_n = 4^{n+1} - (n+2)2^{n+1}.$