VIT®
Vellore Institute of Technology
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Final Assessment Test – April 2026

Course: BAMAT205 - Discrete Mathematics and Linear Algebra

Class NBR(s): 0439/ 0441/ 0443/ 1736/ 1738/ 1740/ 1743/ 1745/ 1748/ 1751/ 1754/ 1756/ 1759/ 1763/ 1765/ 1767/ 1771/ 1774/ 1776/ 1778

Slot: A1+TA1+TAA1

Time: Three Hours

Max. Marks: 100

- KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- DON'T WRITE ANYTHING ON THE QUESTION PAPER

COs	CO Statements
CO1	Apply proof techniques in solving logical problems.
CO2	Solve engineering problems involving counting principles.
CO3	Relate algebraic structures to enhance problem-solving techniques.
CO4	Understand the concepts of vector space, subspaces and linear transformations.
CO5	Apply vector normalization techniques to generate orthonormal basis.

BL – Blooms Taxonomy Level (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create)

Answer ALL Questions
(10 X 10 = 100 Marks)

1. In a software system monitoring application, the system triggers an alert (A) based on the following conditions: [10] CO1 BL2
   - p: CPU usage is high
   - q: Memory usage is high
   - r: Network traffic is high

   System rules:
   a) If CPU usage is high, then an alert is generated.
   b) If memory usage and network traffic are high together, then an alert is generated.
   c) If an alert is generated, then the administrator is notified (N). (i) Write the logical statements for the above rules. (ii) Form the combined propositional expression and (iii) Convert the expression into Disjunctive Normal Form.

2. In a software project management system, consider the following rules: [10] CO1 BL3
   All developers skilled in Java can develop web applications. All developers assigned to project Alpha can develop web applications. Alice is skilled in Java. Bob is assigned to project Alpha.
   (i) Represent the above statements in predicate logic.
   (ii) Using inference rules, prove whether Alice and Bob can develop web applications.
   (iii) Represent the final inference in existential form.

3.a) In a college of 200 students, a survey shows: 120 students use Laptop (L), 100 students use Smartphone (S), 80 students use Tablet (T), 60 students use Laptop and Smartphone, 50 students use Laptop and Tablet, 40 students use Smartphone and Tablet and 30 students use all three devices [10] CO2 BL2

Find:

(i) the number of students who use at least one device.

(ii) the number of students who use exactly one device.

(iii) the number of students who use none of the devices.

OR

3.b) A school has 30 students and 5 different sports: Football, Basketball, Volleyball, Tennis, and Badminton. [10] CO2 BL2

(i) Show that at least 6 students must be interested in the same sport.

(ii) The school wants to form a committee of 4 students such that at least 1 student plays Football. If 10 students play Football, find the number of ways to form such a committee.

4. Use the method of generating function to solve the recurrence relation $a_{n+1} - 8a_n + 16a_{n-1} = 4^n, a_0 = 1, a_1 = 8.$ [10] CO2 BL3

5. Let the generator matrix G of a linear $(6,3)$ code be: [10] CO3 BL2

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

And the received words are $r_1 = 111101, r_2 = 100110, r_3 = 111100,$ $r_4 = 010100$

(i) Find the corresponding parity check matrix H.

(ii) Use H to decode the received code words.

(iii) Find the original messages for all the received messages.

6. Define a relation on the set $A = \{1, 2, 3, 4\}$ which is (i) symmetric, but neither reflexive nor transitive; (ii) reflexive and transitive, but not symmetric; (iii) reflexive and symmetric, but not transitive. [10] CO3 BL3

7. Consider the vectors in $\mathbb{R}^4$: [10] CO4 BL3

$$v_1 = (1,2,0,1), v_2 = (2,5,-1,3), v_3 = (0,1,1,1), v_4 = (3,7,-1,4)$$

(i) Determine whether v_1, v_2, v_3, v_4 are linearly independent.

(ii) Find a basis for the subspace spanned by these vectors.

(iii) Determine the dimension of this subspace.

(iv) Express the vector $w = (4,9,-2,7)$ as a linear combination of the basis vectors.

8. Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$ [10] CO4 BL3

(i) Find the row space, column space, and null space of A.

(ii) Determine the rank and nullity of A, and verify the Rank-Nullity Theorem.

9. Let $u = (1,2,-1)$ and $v = (2,-1,3)$ in $\mathbb{R}^3$. [10] CO5 BL3

(i) Compute the dot product $u \cdot v$.

(ii) Find the lengths of u and v.

(iii) Compute the angle θ between u and v.

(iv) Let $w_1 = (1,0,1), w_2 = (0,1,1), w_3 = (1,1,0)$. Find the matrix representation of the inner product with respect to the basis $\{w_1, w_2, w_3\}$

10.a) Consider the vectors in $\mathbb{R}^3$: $v_1 = (1,1,0), v_2 = (1,0,1), v_3 = (0,1,1)$ [10] CO5 BL3
Use the Gram-Schmidt process to find an orthonormal basis of $\mathbb{R}^3$ from $\{v_1, v_2, v_3\}$.

OR

10.b) Consider the quadratic form: [10] CO5 BL3

$$Q(x_1, x_2, x_3) = 3x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 + 2x_2x_3$$

(i) Express Q in matrix form $Q(\mathrm{x}) = \mathrm{x}^T A\mathrm{x}$.

(ii) Determine the nature of Q: positive definite, negative definite, or indefinite.

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