VIT Vellore Institute of Technol (Deemed to be University under section 3 of the UGC Act,	**Final Assessment Test – Fall 2025-26 Semester - Nov 2026**	
	Course code :BAMAT205	Slot :Common
	Course Title : Discrete Mathematics and Linear Algebra	Time: **Three Hours**
	Class Number(s) :Common	Max. Marks: **100**
	Faculty Name :	School:
	Emp Id :	Mobile No.:

Answer ALL Questions
(10 X 10 = 100 Marks)

Sl. No:	Questions	Module No	Marks
1.	Given the propositional formula $$[(p \to q) \wedge (r \to p)] \to (r \to q),$$ a. Construct the **truth table** and verify whether the formula is a **tautology**. b. Convert the given formula into its **Principal Disjunctive Normal Form (PDNF)**. c. Convert the same formula into its **Principal Conjunctive Normal Form (PCNF)**. d. Comment on the **logical validity** of the statement using inference theory.	1	**10**
2.	In a college, it is observed that every student who studies regularly passes the examination, and anyone who passes the examination eventually gets a job. However, some students do not study regularly, and all such students attend coaching classes. It is further known that any student who attends coaching classes and works hard will pass the examination. At the same time, there exist some students who attend coaching classes but do not work hard. Based on the above information, examine the validity of the following statement: *"There exists a student who gets a job, and there exists a student who does not work hard."* Determine whether the conclusion logically follows from the premises, and justify your answer.	1	**10**
3.	In a survey conducted among 120 students, it was observed that 60 students like Mathematics, 55 like Physics, and 50 like Chemistry. It was further found that 25 students like both Mathematics and Physics, 20 like both Physics and Chemistry, and 15 like both Mathematics and Chemistry, while 5 students like all three subjects. Based on this information, determine the number of students who like at least one of the three subjects, the number of students who like exactly one subject, the number of students who like exactly two subjects, and the number of students who do not like any of the three subjects. **OR** A committee of **6 members** is to be formed from a group of **10 men and 8 women**.	2	**10**

	In how many ways can the committee be formed if it must contain **at least 2 2women and at most 4 men**, and two particular men refuse to serve together on the same committee? Further, if the committee is to be arranged in a row for a photograph, in how many ways can this be done if no two women sit together?		
4.	A sequence $\{a_n\}$is defined by the non-homogeneous recurrence relation $$a_n - 5a_{n-1} + 6a_{n-2} = 2^n, n \geq 2,$$ with initial conditions $$a_0 = 1, a_1 = 4.$$ Using the method of generating functions, obtain the generating function $A(x)$for the sequence and hence find an explicit formula for a_n.	2	**10**
5.	(A) Let Gbe a finite group of order 24. Let Hbe a subgroup of Gsuch that $\lvert H \rvert = 6$. Using **Lagrange's Theorem**, find the number of **distinct left cosets** of Hin G. List all possible orders of subgroups of G. Show that a group of order 24cannot have a subgroup of order 5. (B) Let $(\mathbb{Z}, +)$and $(\mathbb{Z}_{12}, +)$be groups. Define a mapping $$\phi: \mathbb{Z} \to \mathbb{Z}_{12} \text{by} \phi(n) = 4n (\text{mod} 12).$$ Prove that $\phi$is a **group homomorphism**. Determine whether $\phi$is **injective**.	3	**10**
6.	Consider the set $L = \{0, a, b, c, 1\}$equipped with a partial order such that $0$is the least element, $1$is the greatest element, and the elements $a, b, c$are all greater than $0$and less than 1, but are mutually incomparable with each other. verify that $(L, \leq)$is a partially ordered set and draw its Hasse diagram. Determine whether Lforms a lattice. Test whether the distributive laws hold in this lattice, and justify your conclusion with a suitable example.	3	**10**
7.	Let $V = \mathbb{R}^4$and consider the vectors $$v_1 = (1,2,1,0), v_2 = (2,4,2,0), v_3 = (1,3,1,1), v_4 = (0,1,0,1).$$ Determine whether the set $\{v_1, v_2, v_3, v_4\}$is linearly independent. Find a basis for the subspace spanned by these vectors. Hence determine the dimension of the subspace span$\{v_1, v_2, v_3, v_4\}$.	4	**10**
8.	Let $T: \mathbb{R}^3 \to \mathbb{R}^3$be a linear transformation defined by $$T(x, y, z) = (x + 2y + z,\ 2x + 5y + z,\ x + 3y + 2z).$$ Let $B = \{(1,0,1), (0,1,1), (1,1,0)\}$ be an ordered basis for $\mathbb{R}^3$. Verify that $T$is a **linear transformation**. Determine whether $T$is **invertible**. Find the **matrix of $T$with respect to the basis B**. Verify the relation $$[T]_B = P^{-1}AP,$$ where $P$is the change of basis matrix from $B$to the standard basis.	4	**10**
9.	Let $V = \mathbb{R}^4$with the standard inner product. Consider the vectors $v_1 = (1,1,0,0), v_2 = (1,0,1,0), v_3 = (0,1,1,1), v_4 = (1,1,1,1).$ Using the **Gram–Schmidt orthogonalization process**, convert the set $\{v_1, v_2, v_3, v_4\}$into an **orthonormal basis** for $\mathbb{R}^4$. Verify that the resulting vectors are mutually orthogonal and each has unit length.	5	**10**

10.	Consider the quadratic form $Q(x,y,z) = 2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx.$ Express Q in matrix form $X^T AX$, where $X = (x,y,z)^T$. Classify the quadratic form as **positive definite, negative definite, or indefinite**. Justify your answer clearly. **OR** Let $V = \mathbb{R}^3$ be equipped with an inner product defined by $\langle x,y\rangle = 2x_1y_1 + x_1y_2 + x_2y_1 + 3x_2y_2 + x_2y_3 + x_3y_2 + 2x_3y_3,$ for all $x = (x_1,x_2,x_3)$ and $y = (y_1,y_2,y_3)$ in $\mathbb{R}^3$. Let $v_1 = (1,0,1), v_2 = (1,1,0), v_3 = (0,1,1).$ Using this inner product, find the **matrix of inner products (Gram matrix)** of the set $\{v_1, v_2, v_3\}$. Hence compute the **angle between the vectors v_1 and v_2**. Determine whether these two vectors are orthogonal under the given inner product.	5	**10**
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