REG.NO.:

VIT
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF ADVANCED SCIENCES
CONTINUOUS ASSESSMENT TEST - 2
WINTER SEMESTER 2025-2026

SLOT: F2+TF2

Programme Name & Branch	:	B.Tech. - SCOPE
Course Code and Course Name	:	Engineering Physics (BAPHY105)
Faculty Name(s)	:	Anuj Ram Baitha, Samir Ranjan Meher, Rambabu Yalavarthi, Abhinav Anand, Laxmi Narayan Tripathi, Pankaj Sheoran, Kanhaiya Lal Pandey, Sridhar S., Ankush
Class Number(s)	:	VL2025260503669, 3671, 3673, 3675, 3677, 3679, 3681, 3683, 3685.
Date of Examination	:	**22.03.26**
Exam Duration	:	**90 minutes** **Maximum Marks: 50**

General instruction(s):

- Answer All Questions
- Students are permitted to bring any number of textbooks, printouts of e-books; (complete / chapters) and handwritten notebooks (class notes).
- M - Max mark; CO – Course Outcome; BL – Blooms Taxonomy Level (1 – Remember, 2 – Understand, 3 – Apply)
- Course Outcomes (**CO2- Apply** matrix algebra and Dirac notation for the understanding of quantum mechanical problems involving linear operators, eigenvalues and eigenvectors. **CO3- Solve** the particle in a 1-D and 3-D potential box problem using the principles of quantum mechanics.)

Q. No	Question	M	CO	BL
1.	a). Show that $AA^\dagger$ is a Hermitian operator for an arbitrary operator A. b). Verify whether B = $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}$ is a unitary operator or not?	10	CO2	3
2.	Obtain the eigenvalues and normalized eigen vectors of the operator $\begin{pmatrix}2 & -3 & 0\\ 2 & -5 & 0\\ 0 & 0 & 3\end{pmatrix}$.	10	CO2	3
3.	a). State the postulates of quantum mechanics briefly and explain the significance of these postulates in shaping quantum mechanics.	5	CO3	2
	b). A particle of mass "m" traces its path along the X-direction in a "time-independent potential" region. Obtain the equation of motion of the particle in the context of quantum mechanical description.	5		
4.	a). A state of a system is expressed in terms of a set of three orthonormal vectors $\|\alpha\rangle, \|\beta\rangle$ *and* $\|\gamma\rangle$ as $\|\psi\rangle = \frac{\sqrt{2}}{3}\|\alpha\rangle + \frac{a}{\sqrt{3}}\|\beta\rangle + \frac{2}{3}\|\gamma\rangle$. What is the value of "$a$" if $\|\psi\rangle$ is normalised? Calculate the probability of finding the state in each of these states $\|\alpha\rangle, \|\beta\rangle$ *and* $\|\gamma\rangle$.	5	CO3	3
	b). Consider an operator A=$\begin{pmatrix}1 & i\\ -i & 1\end{pmatrix}$ operates on a state vector $\|\psi\rangle$=$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix}$. Determine the expectation value of A in the state $\|\psi\rangle$.	5		

5.	Derive the energy eigenvalues and eigenfunctions of a free particle of mass m confined in a 1D infinite potential well of length $3L$; $V(x) = 0; and\ 0 \leq x \leq 3L$. Show diagrams of the first three energy levels and probability densities of the particle.	10	CO3	2

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Answer Key

<table>
<tr><th>Q. No</th><th>Question</th></tr>
<tr><td>1.</td><td>a). Show that $AA^{\dagger}$ is a Hermitian operator for an arbitrary operator A.

Solution:
$(AA^{\dagger})^{\dagger} = (A^{\dagger})^{\dagger}A^{\dagger} = AA^{\dagger}$ for this step -4M
$AA^{\dagger}$is a self-adjoint and hence Hermitian. -1M</td></tr>
<tr><td></td><td>b). Verify whether B = $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}$ is a unitary operator or not?
Solution:
$BB^{\dagger} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & -i\\ -i & 1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = I;$ for this step -2M
Similarly, $B^{\dagger}B = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = I;$ for this step -2M
Hence, $BB^{\dagger} = B^{\dagger}B = I$; Therefore, B is a unitary operator -1M</td></tr>
<tr><td>2.</td><td>Obtain the eigenvalues and normalized eigen vectors of the operator $\begin{pmatrix}2 & -3 & 0\\ 2 & -5 & 0\\ 0 & 0 & 3\end{pmatrix}$.

Solution:
1. Eigen values (4M)
The characteristic equation is
Det$\begin{pmatrix}2-\lambda & -3 & 0\\ 2 & -5-\lambda & 0\\ 0 & 0 & 3-\lambda\end{pmatrix}$
By solving the above determinant, we get eigenvalues as $\lambda = 3, 1, -4;$ up to here -4M (2 M for writing the characteristic equation, 2M for calculation)
2. Eigen vectors (6M)
For $\lambda = 3$
$$V_1 = \begin{pmatrix}-1 & -3 & 0\\ 2 & -8 & 0\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix} = 0$$
From the above equation $x = 0; y = 0; anb\ z = 1;$
$V_1 = \begin{pmatrix}0\\ 0\\ 1\end{pmatrix}$ -2M
Similarly, for $\lambda = 1$
$V_2 = \begin{pmatrix}3\\ 1\\ 0\end{pmatrix}$ -2M
Similarly, for $\lambda = -4$</td></tr>
</table>

	$V_3 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ **-2M**
3.	a). State the postulates of quantum mechanics briefly and explain the significance of these postulates in shaping quantum mechanics. Solution: For each postulate and its significance **-1M** Five postulates (state of quantum system, Observables, Expectation values, Measurements, and Time evolution) – **5M**
	b). A particle of mass "m" traces its path along the X-direction in a "time-independent potential" region. Obtain the equation of motion of the particle in the context of the quantum mechanical description. Solution: Schrodinger's time-independent wave equation. Writing wave function **-1M** Differentiation and intermediate mathematical steps **-3M** For the equation of motion**-1M** $$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$
4.	a). A state of a system is expressed in terms of a set of three orthonormal vectors $\|\alpha\rangle$, $\|\beta\rangle$ *and* $\|\gamma\rangle$ as $\|\psi\rangle = \frac{\sqrt{2}}{3}\|\alpha\rangle + \frac{a}{\sqrt{3}}\|\beta\rangle + \frac{2}{3}\|\gamma\rangle$. What is the value of "$a$" if $\|\psi\rangle$ is normalised? Calculate the probability of finding the state in each of these states $\|\alpha\rangle$, $\|\beta\rangle$ *and* $\|\gamma\rangle$. Solution: $$\frac{2}{9} + \frac{a^2}{3} + \frac{4}{9} = 1$$ $$\frac{6+3a^2}{9} = 1$$ $$a = 1$$ *For the calculation of a value* ***3M*** Probabilities $$P_\alpha = \|\langle\alpha\|\psi\rangle\|^2$$

	$$P_\alpha = \frac{2}{9}$$ Similarly, $$P_\beta = \frac{1}{3}$$ $$P_\gamma = \frac{4}{9}$$ *For representation of probabilities-**2M***
	b). Consider an operator $A=\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$ operates on a state vector $\|\psi\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Determine the expectation value of A in the state $\|\psi\rangle$. Solution: The expectation value of A is $\langle A\rangle = \langle\psi\|A\|\psi\rangle$ *for this step -**1M*** $A\|\psi\rangle = \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1+i \\ -i+1 \end{pmatrix}$; *for this step **2M*** $\langle A\rangle = \frac{1}{\sqrt{2}}(1 \quad 1) \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} 1+i \\ -i+1 \end{pmatrix} = \frac{1}{2}(1+i-i+1) = 1;$ *for this calculation -**2M***
5.	Derive the energy eigenvalues and eigenfunctions of a free particle of mass m confined in a 1D infinite potential well of length $3L$; $V(x) = 0;\ and\ 0 \le x \le 3L$. Show diagrams of the first three energy levels and probability densities of the particle. Solution $$V(x) = \begin{cases} 0; 0 \le x \le \\ \infty; otherwise \end{cases}$$ $\frac{\partial^2\psi}{\partial x^2} + k^2\psi = 0; where\ k^2 = \frac{2mE}{\hbar^2}$ (*up to here **1M***) Solution for the above equation $\psi(x) = A\sin kx + B\cos kx$ Applying boundary conditions At $x = 0; B = 0$ At $x = 3L;$ $$\psi(3L) = 0 = A\sin(3kL) = 0$$ $$3kL = n\pi$$ $$k = \frac{n\pi}{3L}; n = 1,2,3 \ldots.$$ Therefore $E_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{3L}\right)^2 = \frac{n^2\pi^2\hbar^2}{18mL^2}; n = 1,2,3 \ldots.$ (*up to here **4M***) Wavefunction normalization $$\int_0^{3L} \|\psi(x)\|^2 dx = 1$$ By solving, we get $A = \sqrt{\frac{2}{3L}}$

So, $\psi_n(x) = \sqrt{\frac{2}{3L}} \sin\left(\frac{n\pi x}{3L}\right)$ *(up to here, **3M**)*

*For diagrams **2M** each carries **1M***